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PARAMETER ESTIMATING STATE RECONSTRUCTION

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Office of The Associate Director for Engineering

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16. ABSTRACT

The focus of this research is parameter estimation for systems whose entire state cannot be measured. Linear observers are designed to recover the unmeasured states to a sufficient accuracy to permit the estimation process. These systems must be observable. There are three distinct dynamics that must be accommodated in the system design: the dynamics of the plant, the dynamics of the observer, and the system updating of the parameter estimation. The latter two are designed to minimize interaction of the involved systems.

These techniques are extended to weakly nonlinear systems. The application to a simulation of a Space Shuttle POGO system test is of particular interest. A nonlinear simulation of the system is developed, observers designed, and the parameters estimated.

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LIST OF SYMBOLS

Symbol	Definition
A	system Jacobian or state matrix (n x n)
A_1	pump constant $(\text{in.}^2)^{-1}$
$a \leqslant b$	a less than or equal to b
B	control distribution matrix (n x q)
B_1	pump constant $(\text{rpm})^{-1}$
C_A	accumulator compliance $(\text{in.}^2)^{-1}$
C_B	pump compliance $(\text{in.}^2)^{-1}$
C_D	duct compliance $(\text{in.}^2)^{-1}$
C^T	measurement matrix (m x n)
CTL-V	Santa Suzanna Test Stand
D	Jacobian estimate matrix
D_i	dynamic systems
D'_1	linearized head rise constant (in.^2)
D'_2	linearized head rise constant $(\text{sec})^{-1}$
DW_A	accumulator weight flow rate (lbf/sec)
DW_{FL}	feedline weight flow rate (lbf/sec)
DW_{OP2}	weight flow rate at the orifice (lbf/sec)
DW_{OS}	pump and duct weight flow rate (lbf/sec)
$E' \times E^n$	definition of a space
e(t)	error, $x(t) - \hat{x}(t)$

LIST OF SYMBOLS (Continued)

Symbol	Definition
$e^{(\ )}$	natural number to ( ) exponent
F	state matrix of auxiliary system
$F_{(\ )}$	linearized flow (lbf/sec)
f(x,t)	nonlinear state relation to the state derivative
GOX	gaseous oxygen
G′(T)	discrete transition matrix for observer
G(T)	discrete state transition matrix
$G_1(T)$	estimate of the transition matrix
$H(x(t_0))$	observability mapping
H	mapping of state into the auxiliary state
H(T)	discrete control distribution matrix
$H_1(T)$	estimate of the distribution matrix
H′	pump head rise
h(t,x)	nonlinear system measurements
$in.^2$	square inches
K	observer gain matrix (n x m)
k	kth interval
LDE	linear differential equation
LOX	liquid oxygen
L(T)	discrete distribution matrix of state
L′(T)	discrete distribution matrix of control to observer

LIST OF SYMBOLS (Continued)

Symbol	Definition
$L'_1(T)$	discrete estimate of distribution matrix of control to observer
L_A	accumulator inertance $(in.^2/sec)^{-1}$
L_D	duct inertance $(in.^2/sec)^{-1}$
L_L	line inertance $(in.^2/sec)^{-1}$
L_N	discharge section inertance $(in.^2/sec)^{-1}$
ℓ	ℓth term
m x n	matrix size, m rows and n columns
NDE	nonlinear differential equation
O_{mk}	null matrix (m x k)
P	transformation relating x and z
POGO	fluid mechanical dynamic coupling of large space vehicles
P_A	accumulator pressure (psi)
P_{OD1}	pump discharge pressure (psi)
P_{OI2}	duct pressure (psi)
P_{OP2}	pressure past the orifice (psi)
P_{OS}	pressure at the pump inlet (psi)
P_T	supply pressure (psi)
psi	pounds per square inch
R_A	accumulator resistance $(lbf/in.^2)^{-1}$
R_D	duct resistance $(lbf/in.^2)^{-1}$
R_L	line resistance $(lbf/in.^2)^{-1}$

LIST OF SYMBOLS (Continued)

Symbol	Definition
R_N	discharge section resistance $(lbf/in.^2)^{-1}$
rpm	revolutions per second
S	Laplace operator
S_{O1}	pump speed (rpm)
sec	seconds
T	sample period (sec)
t	time (sec)
t_0	initial time (sec)
U	control signal (q dimensional)
x	state vector (n dimensional)
x(k)	kth value of the discrete state
x_0	initial state
x_1	model state vector (n dimensional)
y	measurement vector (m dimensional)
z	auxiliary state
Δ	linearized variation
$\underline{\Delta}$	defined as
Δ_i	ith principle minor
$\delta(\ )$	variation of ( )
ϵ	an element of, or an arbitrary small number

LIST OF SYMBOLS (Concluded)

Symbol	Definition
$\Gamma_{POP1}(\ )$	pump head rise characteristics
$\Phi(t,t_0)$	state transition matrix
Φ_{OP1}	dimensionless pump parameter
Ω	solution set in the state space
τ	dummy integration variable
$\int$	integral
$\rightarrow$	mapping
!	factorial
$((\ ):(\ ))$	partitioned matrix
$(\dot{\ })$	time derivative
$(\hat{\ })$	estimate of ( )
$(\ )(t)$	time dependence
$(\ )_{ij}$	elements of the matrix ( ), i-row, j-column
$(\ )^k$	exponent, kth power
$(\ )^{(i)}$	ith time derivative
$(\ )^T$	matrix transpose
$(\ )^{-1}$	matrix inverse

The advent of large-scale computing equipment has made these concepts feasible. A logical extension of the technique considered here would be to put an additional loop around the object dynamic system. This loop would determine a model structure with which to estimate parameters. These parameters would permit state determination. This would approach the advanced objective previously mentioned. This research will be confined to those cases for which a rational model structure has been chosen.

To perform a linear analysis, a mathematical model structure must be contrived or selected. Many considerations contribute to the selection of a rational model structure. The first step is to define a response matching criterion and then choose a model structure capable of fulfilling the criterion. This fulfillment, generally checked by simulation, is accomplished by time response comparison, frequency response comparison or, more generally, a combination of both. The inclusion of small signal nonlinearities, such as stiction, windup, hysteresis, and deadband, is dictated by the dynamic effect on the system as ascertained by the response matching criterion. A central feature of model structure selection is system dimensionality truncation. Dimensionality truncations for model selection are of two types: those due to modeling complexity, and those to reduce dimensionality of an already chosen model structure. Modeling complexity is necessarily broken at some level since most complex system models could be made infinite in extent. These truncations are made on the basis of insight, feel, experience, and logistics of the computational equipment available. A rational choice of model structure may simply be the exclusion of dynamic effects in some frequency regime for which the control and/or the plant are nonresponsive. A more complicated scheme consists of including only coupling dynamics. The implementation of the coupling dynamics scheme is straightforward although sometimes computationally difficult. A subcomponent representation is determined first as an isolated system and then compared to the subcomponent representation in the closed-loop system. If the pole-zero representation moves more than some judgmental amount, that subcomponent must be included in the overall system dynamics. If the subcomponent's dynamics may be discarded, the steady-state contribution is accounted for algebraically. A simple state variable criterion is to eliminate those states whose derivatives remain less than some judgmental amount. Many other schemes may be devised as well as combinations of these schemes. The ultimate criterion is the satisfactory working of the finally designed and analyzed system.

The purpose of the research may be summarized as providing the "best" representation of the system linear model, or Jacobian, for a given configuration. Best representation means the best model attainable under a qualitative judgment involving accuracy, measurement inaccuracies, and system disturbance. The principal objective is the assessment of the validity of the mathematical model used to design a subject system.

While there are other techniques providing the same information, the proposed technique reduces the system history required for solution. Most estimation techniques avoid the partially measured state vector because the standard approach is to adjoin the unmeasured states to the parameter matrix. The result is that an n-squared problem has

CHAPTER I
INTRODUCTION

Statement of the Problem and Objectives

An advanced objective of control system theory is to build a learning device so an unknown system is directed to some goal by the device. First the identification of the system is implemented. Based on the identification, the state of the system is determined. An appropriate control stimulus results in a response that satisfies some rational figure of merit. This is presently accomplishable in only basic systems [1]. The purpose of this research is to develop and demonstrate a technique, consisting of several concepts, that permits simultaneous calculation of the state and a reasonable facsimile of the plant. These concepts center upon state reconstruction and parameter estimation.

There is a certain amount of literature using these concepts [2,3], but only recently have the combined concepts of state reconstruction and parameter estimation been exploited [4] to provide information simultaneously of the state and the system representation. The literature is confined to linear autonomous systems, while this research will attempt to extend the developed techniques to nonautonomous and nonlinear systems. The technique of Reference 4 requires a Liapunov function of the unknown system, which is possible for linear systems. The advantage is the synthesis of a globally convergent scheme. The disadvantages are that the Liapunov function may not exist for nonautonomous and nonlinear systems. The method of this research uses a steepest descent of gradient type method. A disadvantage of a gradient method is that initial estimates must be close. However, in practice, the system is reasonably well known and this disadvantage is not overwhelming.

This paper will attempt to apply the combined techniques of parameter estimation and state reconstruction to the measurements of nonlinear physical systems. In practice, linear systems do not exist. However, there are regions of operation on which any system exhibits nearly linear behavior. The limitation may exist that the region of linearity is too small or that expected excitations will drive the system out of its linear region. In any case, a comprehensive control or system analysis begins with a linearization of the subject system. For many applications, the analysis either forms a basis for design or provides a rationale for redesign or alteration of the system. A fundamental, but sometimes unanswered, question is "how good a representation of the system is this linear model?" In many fields and applications, an a posteriori analysis is undertaken to assess the mathematical modeling accuracy. Generally, this consists of a manual iterative assessment until some degree of accuracy is achieved.

Parameter estimating state reconstruction can be applied, as a black box, to a system's measurements to provide a real-time assessment of the current linear model of that system. By assessing the response from different pieces of hardware, statistics may be compiled as to the spread of that system's operation. An analysis of extreme conditions provides an assessment of the sensitivity of the system to a real environment. As a result, a quantitative assessment of the system design analysis is possible.

been expanded to an n-squared plus n problem. On the other extreme, the Liapunov approach becomes difficult for systems of greater than single output because a Liapunov function must be contrived for each of the outputs which also are coupled. The proposed technique retains the n-squared dimensionality of totally measured systems.

Figure 1-1 is a diagram of the system description. A state reconstructor is used to recover the unmeasured states. The state reconstructor is used with the reference model to form an error for the estimation process. Establishing validity of using the state reconstructor output and addressing coupled systems dynamics are the principal concerns to be analyzed.

Chapter Description

Chapter I is a delineation of the area of research with an identification of problem areas and overall objectives. Chapter II will develop the observability theorems for the systems to be considered. This will be followed by developing and presenting the necessary observer or state reconstruction theory to support the research. The treatment will be subdivided to treat linear time invariant, linear time varying, and nonlinear systems. Chapter III will address parameter estimation with state reconstruction. The problem of model structure and model matching criteria will be analyzed.

Chapter IV will develop a dynamic model of the CTL-V Space Shuttle POGO test facility to be analyzed by the technique of this paper. CTL-V is a particularly good example since fully half of the involved parameters are unmeasurable. A nonlinear model will be developed and the linearized equivalent will be analyzed dynamically at the rated power level operating point. Chapter V will design the necessary observers, to permit application of the technique, and present the results of the simulation and analysis. Finally the technique will be summarized considering its advantages, disadvantages, and unique characteristics.

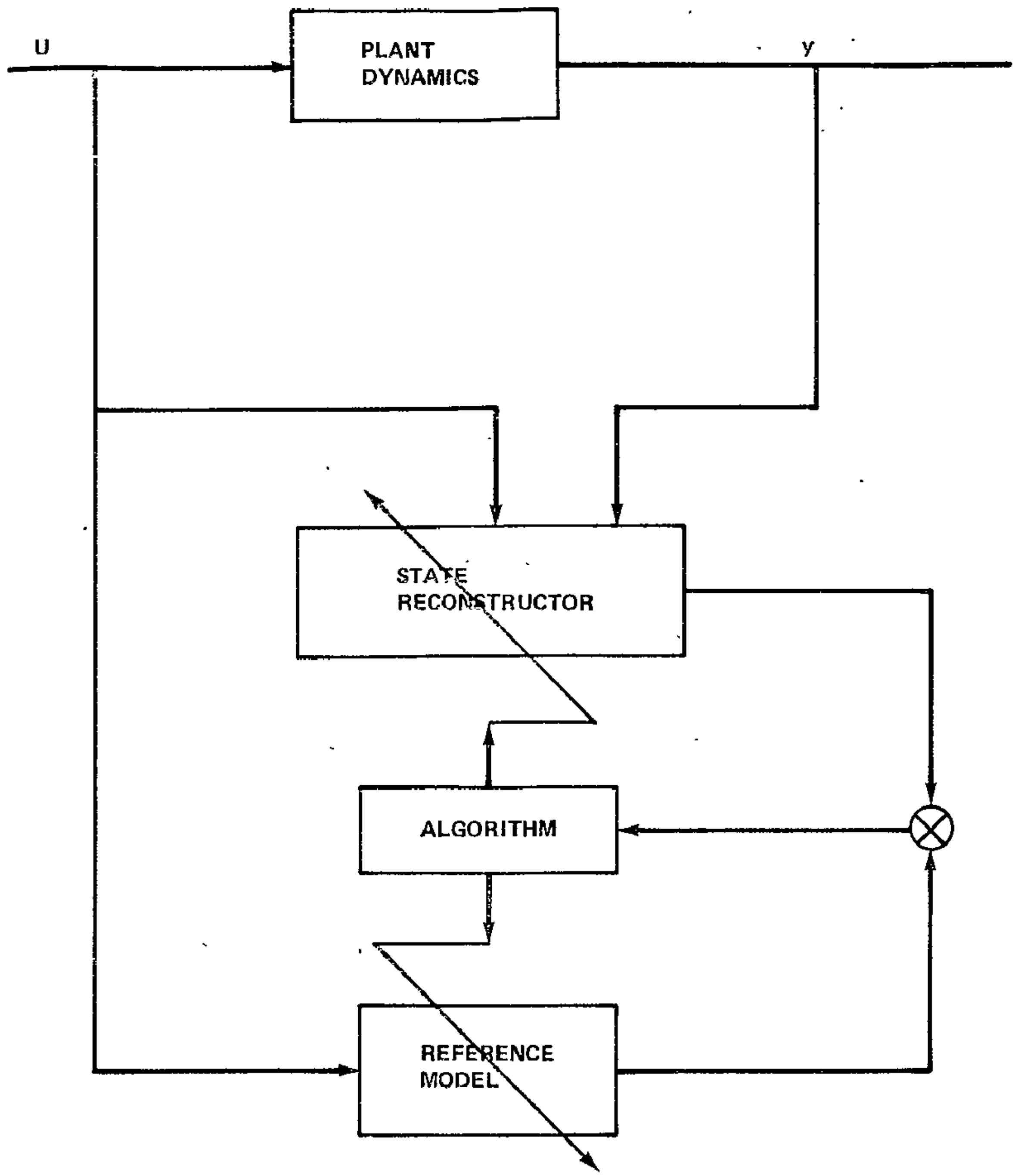

Figure 1-1. Block diagram of the system.

CHAPTER II
OBSERVABILITY AND OBSERVERS

Paramount to any closed-loop control consideration is the measurability and/or observability of the object dynamic system. An unobservable system may be controllable only in an open-loop sense. Therefore, this research will be confined to closed-loop control and thus observable systems. Observability will be dealt with in detail, particularly those aspects pertaining to linear constant coefficient, linear time varying, and nonlinear dynamic systems. Observability is a required condition for the state reconstruction process and is included for completeness.

Many unobservable systems may be recast in a form tractable to the technniques of this research. The procedure consists of partitioning the system into observable and unobservable parts. The partitioned observable part may then be handled as an observable system. If the unobservable partition interacts with the observable partition, the interacting elements may be treated as disturbance inputs to the observable system. The partitioning process can be accomplished by means of a transformation to controllability-observability canonical form [5].

A pedagogic examination of the relationship between controllability and observability will aid in the development of the requirements for observability. If x is an n-space representation of the system, y is an m-space representation of the measurements of that system, and x_0 is the initial state, then the controllability problem may be defined as the existence of a solution from x_0 to a desired state, x_f. The observability problem is defined as the existence of a unique one-to-one mapping from x to y. This has been elucidated by Kalman [5] as the principle of duality. The principle of duality [6] depends on the uniqueness of the solution and the mapping. This principle applies to linear constant coefficient and linear time varying systems. However, for nonlinear systems, existence is not necessarily uniqueness and the principle does not apply [7]. It remains to develop the conditions for assuring a unique one-to-one mapping from x to y for the various dynamical systems.

Observability Theorems

Theorem: (Linear time invariant)

The system

$$\dot{x} = Ax$$

$$y = C^T x \quad ,$$

where x is an n vector, y is an m vector ($m \leqslant n$), A is n x n, and C^T is m x n, is completely observable if and only if the composite n x mn matrix

$$[C, A^T C, \ldots A^{T^{n-1}} C]$$

is of rank n. The proof of the preceding is given in many texts [6,8].

Theorem: (Linear time varying)

The system

$$\dot{x} = A(t)x$$

$$y = C^T(t)x$$

where the variables are as previously defined, is completely observable on the time interval $t_0 \leqslant t \leqslant t_1$ if and only if the matrix

$$M(t_0,t_1) = \int_{t_0}^{t_1} \Phi^T(\tau,t_0)\, C(\tau)\, C^T(\tau)\, \Phi(\tau,t_0)\, d\tau$$

is nonsingular. The matrix $\Phi(t,t_0)$ is the unique fundamental matrix satisfying

$$\frac{d}{dt}\Phi(t,t_0) = A(t)\,\Phi(t,t_0) \quad , \quad \Phi(t_0,t_0) = I_n$$

For complete observability, the above must hold for every t_0 and some finite $t_1 > t_0$. The proof of the preceding theorem is likewise found in most modern control texts [6,8].

For nonlinear systems, a more precise definition of terms is required because existence and uniqueness are no longer equivalent. The nonlinear system may be represented as

$$\dot{x} = f(t,x)$$

$$f:[t_0,t_1] \; x\Omega \subset E' \times E^n \rightarrow E^n \tag{2-1}$$

with measurements

$$y = h(t,x)$$

$$h:[t_0,t_1] \; x\Omega \subset E' \times E^n \rightarrow E^m \tag{2-2}$$

The initial state $x(t_0)$ is in general unknown since $m \leqslant n$. Now assume that the ℓth order derivatives of f and h exist for every $x \epsilon \Omega$ and for every $t \epsilon [t_0,t_1]$ where $\ell m \geqslant n$. Expand y(t) as a Taylor series

$$y(t) = y(t_0) + \dot{y}(t_0)(t - t_0) + \ldots + \frac{y^{(\ell)}(t_0)}{\ell!} (t - t_0)^\ell \tag{2-3}$$

where

$$y(t_0) = h(x(t_0), t_0) \triangleq h_0(x(t_0), t_0)$$

$$\dot{y}(t_0) = \frac{\partial h_0}{\partial t} (x(t_0),t_0) + \left(\frac{\partial h_0}{\partial x_0} (x(t_0), t_0) \right) f(x(t_0),t_0) \triangleq h_1(x(t_0),t_0) \quad .$$

Now define

$$H(x(t_0))$$

where

$$z = \begin{bmatrix} y(t_0) \\ \cdot \\ \cdot \\ \cdot \\ y^{(\ell-1)}(t_0) \end{bmatrix} \quad H(x(t_0)) = \begin{bmatrix} h_0(x(t_0),t_0) \\ \cdot \\ \cdot \\ \cdot \\ h_{\ell-1}(x(t_0),t_0) \end{bmatrix} \tag{2-4}$$

The nonlinear map $H(x(t_0))$ is called the "observability mapping" of the system. The system described by Eqs. (2-1) and (2-2) is said to be completely observable in Ω_0 on the time interval $[t_0,t_1]$ if there exists a one-to-one correspondence between the set Ω_0 of initial states and the set of trajectories of the observed output $y(t)$ for $t\epsilon[t_0,t_1]$. If the observability map H is one-to-one Ω_0 to $H(\Omega_0)$, then knowing z uniquely determines $x(t_0)$ so that the system is completely observable. Several publications [7,9] have investigated these conditions for global observability.

Theorem:

The system described by Eqs. (2-1) and (2-2) is completely observable in the set Ω_0 of initial states on the time interval $[t_0,t_1]$ if

(1) $\ell m = n$, where n is the span of the state, m is the number of outputs, and ℓ is the ℓth derivative of f and h which are assumed to exist.

(2) The observability mapping of this system is differentiable.

(3) There exists an $\epsilon > 0$ such that the absolute values of the leading principal minors $\Delta_1, \Delta_2, \ldots, \Delta_n$ of the system Jacobian satisfy

$$|\Delta_1| \geqslant \epsilon \quad , \quad \frac{|\Delta_2|}{|\Delta_1|} \geqslant \epsilon \quad , \ldots , \quad \frac{|\Delta_n|}{|\Delta_{n-1}|} \geqslant \epsilon$$

for all $x\epsilon E^n$, then H is one-to-one from E^n onto $H(E^n)$. This result is proven in Reference 9.

The development of the conditions for nonlinear observability gives visibility to a minor theorem that can be applied to linear systems. If the observability mapping is related to linear systems, it reduces to the familiar form of the condition for observability. Another use may be made of these results. First, consider the case where there is only one measurement. Now the vector z is simply

$$z = \begin{bmatrix} y \\ \dot{y} \\ \cdot \\ \cdot \\ \cdot \\ y(n-1) \end{bmatrix} = \begin{bmatrix} C^T \\ C^T A \\ \cdot \\ \cdot \\ \cdot \\ C^T A^{n-1} \end{bmatrix} x \qquad (2\text{-}5)$$

which is the transpose of the observability matrix. This matrix must be of rank n since the system is observable so that premultiplying by the transponse and taking the inverse, Eq. (2-5) may be written as

$$x = \left[\begin{bmatrix} C^T \\ C^TA \\ \cdot \\ \cdot \\ \cdot \\ C^TA^{n-1} \end{bmatrix}^T \begin{bmatrix} C^T \\ C^TA \\ \cdot \\ \cdot \\ \cdot \\ C^TA^{n-1} \end{bmatrix}\right]^{-1} \begin{bmatrix} C^T \\ C^TA \\ \cdot \\ \cdot \\ \cdot \\ C^TA^{n-1} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \cdot \\ \cdot \\ \cdot \\ y^{(n-1)} \end{bmatrix} \quad . \tag{2-6}$$

Given the output and n-1 derivatives of the single output, the n-state vector may be deterministically obtained. This result is of little use because, in practice, it is difficult to differentiate the measurement with great insensitivity or precision.

Observers

Observer or state reconstructors have been repeatedly examined in the literature since Luenberger [10,11] quantified the concept. The reason for this interest was the advent of state variable theory [6], which organized a dynamic system in such a fashion that the observations of system are not necessarily the measure of the system. The system may be made up of n states and observed by m observations where n is not necessarily equal to m, but generally is greater than m. The observer fills a need to have n states from m observations.

The objective of the observer process is to provide reasonable approximations to those states that are not directly measured. Then these states are available for use in the implementation of a control law or strategy. Observers also find use in system estimation and identification. The conceptual basis for the observer lies in the process of driving an auxiliary dynamic system with the available outputs of the subject dynamic system.

The state reconstructor is an auxiliary dynamic system that deterministically calculates the states using the difference between the real measurements and the measurements from the reconstructor. The state reconstructor is an intriguing mathematical phenomenon because apparently "free information" is acquired. That is, m measurements are sufficient to determine an n-vector state. Considerable attention has been focused on "reduced order observers." If some of the states are directly observed the system may be partitioned to form two related systems. These related systems consist of one measured and the other reconstructed.

One of the most significant problems of observers is knowing initial conditions for the reconstructor system. The initial conditions of the measurements are apparent but these are not necessarily the states. This problem is complicated by errors in the system

parameters. A solution to this dilema is the "robust observer" that will be investigated further. These observers have the property of converging to the proper state even though the differential equation coefficients are not accurately known. This is accomplished in a manner analogous to integrating out steady-state error.

Observer Development

An auxiliary dynamic system will almost always serve as an observer in that its state will tend to follow a linear transformation of the subject dynamic systems state. The design of the observer consists of incorporating that linear transformation into the process, thus providing an immediate and direct measure of the state.

Let D_1 be a free dynamic system describable by

$$\dot{x}(t) = Ax(t) \tag{2-7}$$

and D_2 will be the auxiliary dynamic system of the form

$$\dot{z}(t) = Fz(t). \tag{2-8}$$

This auxiliary system will be driven by the outputs of Eq. (2-7)

$$y(t) = C^T x(t) \tag{2-9}$$

so that

$$\dot{z}(t) = Fz(t) + Hx(t) \tag{2-10}$$

where

$$H = KC^T \tag{2-11}$$

in which K is a gain matrix selected to achieve some goal. Now

$$\dot{z}(t) - P\dot{x}(t) = Fz(t) + HPx(t) - PAx(t) \tag{2-12}$$

If

$$H = PA - FP \tag{2-13}$$

then

$$\dot{z}(t) - P\dot{x}(t) = F(z(t) - Px(t)) \tag{2-14}$$

which has

$$z(t) = Px(t) \tag{2-15}$$

as a solution, demonstrating the assertion of the preceding paragraph. Notice that D_1 and D_2 need not have the same dimension.

This suggests the "identity observer" where the transformation P is the identity matrix. For this type of observer, D_1 and D_2 must be the same dimension. Note that z(t) becomes an estimate of x(t), F becomes A-H, and Eq. (2-10) may be rewritten as

$$\dot{z}(t) = \dot{\hat{x}}(t) = (A - KC^T)\,\hat{x}(t) + KC^T x(t) \quad . \tag{2-16}$$

Let the error between x(t) and $\hat{x}(t)$ be defined as e(t). Now,

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= Ax(t) - A\hat{x}(t) + KC^T\hat{x}(t) - KC^T x(t) \\ &= (A - KC^T)(x(t) - \hat{x}(t)) \\ &= (A - KC^T)\, e(t) \end{aligned} \tag{2-17}$$

which expresses the dynamics of Figure 2-1.

If C^T and A are real matrices, then the eigenvalues of $A - KC^T$ can be made to correspond to the set of eigenvalues of any n x n real matrix by suitable choice of K if and only if (C^T,A) is completely observable. This has been proven in the literature several times, notably by Gopinath [12]. This implies that e(t) may be driven to zero arbitrarily fast by suitable choice of eigenvalues of the augmented system $A - KC^T$. The response is normally dictated by a trade between accuracy and performance. If the eigenvalues are made extremely large negative, the system tends to act as a differentiator and is highly sensitive to noise and other disturbances. Some of these effects will be demonstrated with a subsequent example.

The observer is easily expressed as difference equations. Equation (2-7) becomes

$$x(k + 1) = G(T)\, x(k) + H(T)\, U(k) \tag{2-18}$$

where $G(T) = e^{AT}$, and Eq. (2-16) becomes

$$\hat{x}(k + 1) = G'(T)\, \hat{x}(k) + L(T)\, x(k) + L'(T)\, U(k) \tag{2-19}$$

where $G'(T) = e^{(A-KC^T)T}$

$$L(T) = \int_0^T e^{(A-KC^T)\tau} KC^T \, d\tau \tag{2-20}$$

and

$$L'(T) = \int_0^T e^{(A-KC^T)\tau} B \, d\tau \tag{2-21}$$

This may be represented as in Figure 2-2. If the system is not totally observable, then the ability to place eigenvalues is restricted. In fact, some of the errors may be unbounded. This does not imply a lack of system controllability but rather a lack of adequate control command to the state reconstructor. Only if the system is totally observable can the eigenvalues of the error system be arbitrarily placed. Without total observability, the reconstructor will be uncontrollable.

Let M be the nonsingular transformation that takes the system to observable canonical form:

$$x = Mz \qquad (2\text{-}22)$$

Rewriting Eq. (2-16)

$$M\dot{\hat{z}} = AM\hat{z} + BU + K(C^T Mz - C^T M\hat{z}) \qquad (2\text{-}23)$$

$$M\dot{z} = AMz + BU \qquad (2\text{-}24)$$

$$e = Mz - M\hat{z} \qquad (2\text{-}2$$

$$\dot{e} = M\dot{z} - M\dot{\hat{z}} \qquad (2\text{-}26$$

$$\dot{e} = (AM - KC^T M)\, M^{-1} e \qquad (2\text{-}27)$$

But

$$C^T M = (C^{T'} : O_{mk})$$

where $C^{T'}$ is m x n-k where k states are unobservable, and $KC^T M$ is n x n-k. It is apparent that the last k columns of AM are unaffected by choice of K

Nonlinear observers have been developed for several cases [13,14,15]. In general, these observers are highly system dependent, and are very sensitive to initial conditions and gains. System dependent means that the closed form observer may be developed only for distinct classes of nonlinear systems. Further restrictions are the conditions required for one-to-one mappings which assure observability. More general realizations of observers, characterized by Reference 13, require nonlinear gain schedules for convergence and limitations on initial conditions.

Observers for Use in Estimation

The principal problems in applying observers to estimation are isolating the dynamics of the system from the dynamics of the observer and knowing the matrix A. The observer existence is based on some knowledge of the matrix A. While in many cases

the accuracy of the reconstruction process is independent for small errors of the precision with which A is known, there are cases where the observer will diverge [16]. Due to manufacturing tolerances, material acquisition, sensor accuracy, modeling truncation, and a myriad of other reasons, the A matrix will never be precisely known. The concept of an observer is analogous to a pole-zero canceling compensation so that sensitivity is an inherent design problem.

Observers of rank less than n are known as reduced order observers. The unreconstructed states are obtained directly from measurements and the system is partitioned to separate the directly measured states from the rest. The remaining states are recovered by an observer of order n minus the number of measured states. The minimal observer results when all the measurements are used to identify specific states. However, the minimal observer results in a totally open-loop observer for the unmeasured states. Philosophically, reduced order observers are attractive due to the reduced dimension. In practice the reduced order observer only simplifies the observer design. This simplification is easily outweighed by certain advantages of the identity observer. A minor theorem [17] shows that any identity observer is robust. Battacharyya [17] defines a robust observer as a closed loop system, closed on the error between plant and estimate, and one that possesses redundancy. Measurement redundancy means that, implicitly or explicitly, at least one linear combination of states is contained in the measurements.

These recent studies [16,17] have been directed to the sensitivity of observers. While the emphasis of these studies has been on reduced order observers, the sensitivity results generally apply and will be used as justification for assertions and assumptions of this research. The primary assertion is, if a robust observer is designed, the estimated states will converge asymtotically to the states even if errors exist in the estimate of the plant parameters. The use of identity observers removes the concern that the observer is not robust. Further the observers will be designed so the augmented system eigenvalues are critically damped. This stipulation is of little value for linear systems not disturbed by random inputs. However, for weakly nonlinear and perturbed systems, intuition and experience indicate the critically to highly overdamped roots will behave in a superior manner.

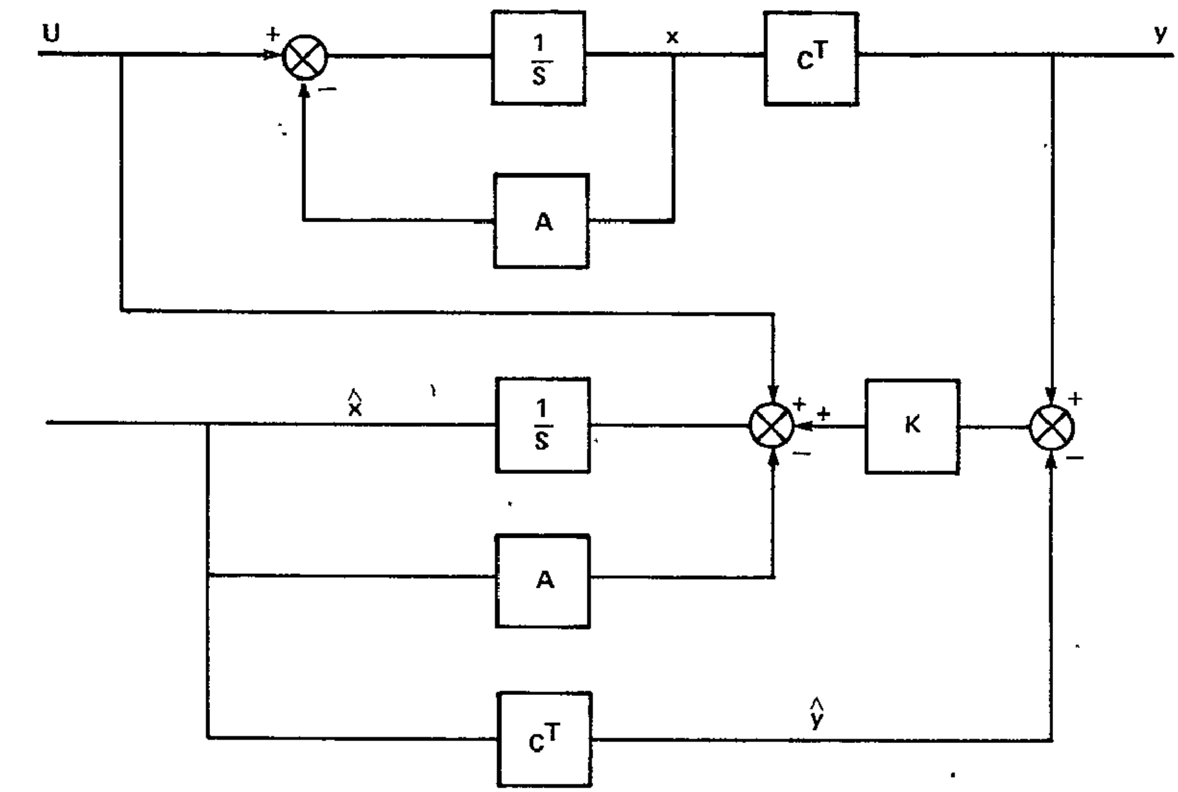

Figure 2-1. Identity observer.

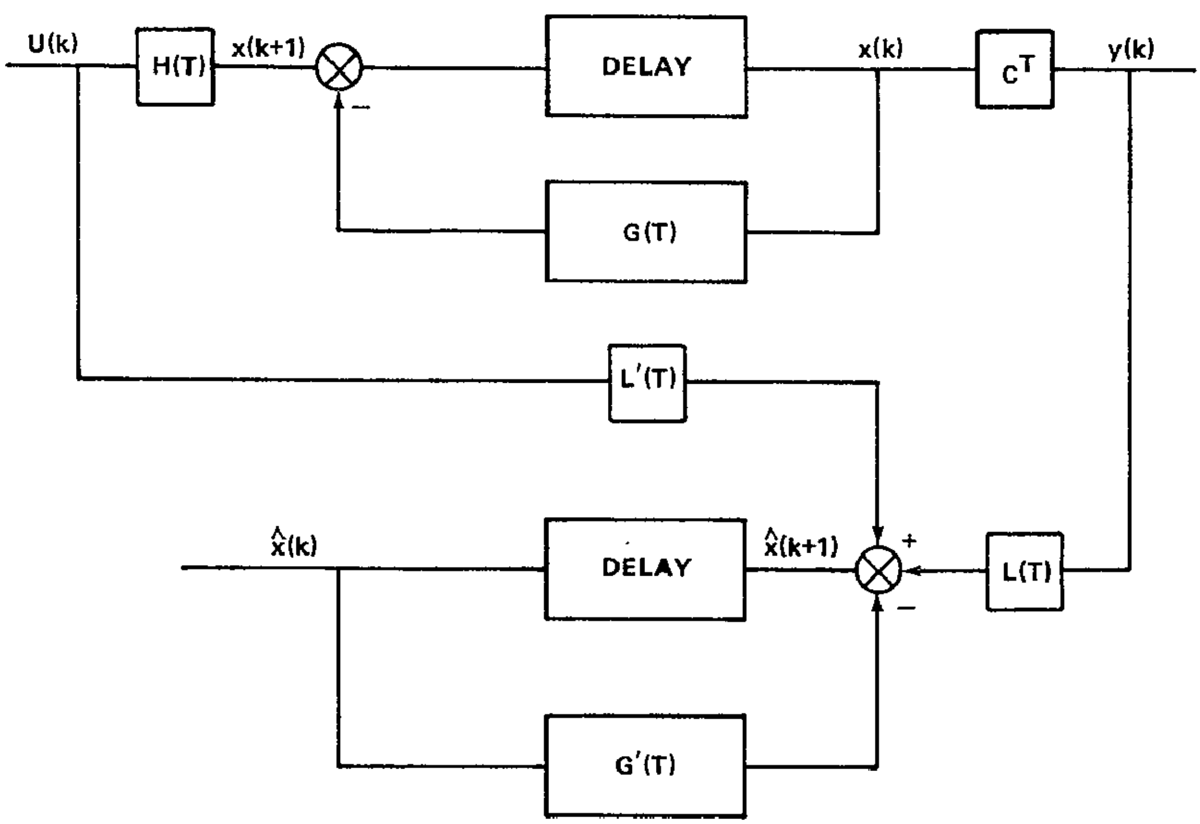

Figure 2-2. Difference equation form of the identity observer.

CHAPTER III
PARAMETER ESTIMATING STATE RECONSTRUCTION

Complex multiloop system analysis and control design are most generally predicted on a linear analysis. Sufficient mathematical modeling is developed to assure a reasonable facsimile of the physical process, at least in the region of interest. Therefore, it is highly desirable to quantify differences between behavior, in the small, of the process and its analysis linear model. This quantification provides a final step in the design cycle, and determines expected variations from the process description used in the system design. These variations may be sufficiently large to dictate redesign of the controls or of the process itself. Unfortunately, all states are generally not available through measurement, so that the process of determining the best linear model is coupled with determining the states on a time history basis. The basic assumptions of this chapter are: that a model structure has been determined, that the system inputs are known, that there is no input disturbance or measurement noise, and that measurement of the state is incomplete.

Estimation of Partially Measured Systems

There are two alternatives available to solve the problem of estimating the parameters of a system whose states are not all available through measurement. The first is to augment the parameter estimation problem with the unmeasured states. The second is to develop in-line estimation algorithms which permit the simultaneous calculation of the desired parameters and states. The former results in an increase in dimensionality of the problem. In general, the parameter estimation problem is an n-squared problem. Augmentation can raise this to as high as n-squared plus n. This causes a long data stream to be required in the claculations and may result in convergence problems due to the age of oldest data. The latter approach is likewise potentially an n-squared plus n problem, but the data stream is the same as for a fully measured estimation problem. There are several methods available for the calculation of the states. The most straightforward approach is the state reconstructor or observer.

Gates [13] points out that the use of standard linear observers causes problems due to the coupling dynamics between the state reconstruction process and the plant estimation process. These problems are of particular concern if the dynamics of the plant are fast. This robs the designer of flexibility in locating system poles and zeroes to better solve the problem. Some of the difficulties of observers under variations have been addressed by Battacharyya [16]. However, if the system is stable the robust observer may be designed. Robust observers are stable and converge to the actual states even if the system differential equations are not accurately known. The reason for this convergence to the actual state is quite analogous to an integrator in the feedback of a control system. The steady-state error is driven to zero.

If the system dynamics are reasonably fast and if the plant is stable, the design approach is straightforward. The approach may be described as a periodic sequential estimation process. This is implemented as an estimation update periodically occurring after the settling time of the state reconstructor has expired. This process may be viewed as a sequence of state reconstruction problems with the error in parameter estimate forming the initial condition guess for the subsequent state reconstruction. The sequence may be designed to provide a set of essentially uncoupled state reconstruction problems, each with a successively better estimate of the differential equation coefficients. For example, if the system has a 0.1 sec settling time, an observer with a settling time of 1 sec is uncoupled from the system and generally four or five iterations provide reasonable convergence. Therefore in this example, the estimation process may be completed in 4 to 5 sec. There is inherent design flexibility to adjust these settling times (of observer and estimator) for a desirable response.

This approach appears more desirable than the nonlinear observer-like system proposed by Gates. The design of Gates' observer is much more system dependent than a system tuning approach. This is undesirable because sufficient information concerning the system may not be available. A further, and more serious, disadvantage of Gates' observer is that a stable configuration may not in fact exist.

The problem is then to keep the n-squared estimation problem while developing simultaneously the system states. Consider a robust observer compared with a best system estimate or system model. If the systems are adjusted so that the overall computation cycle is longer than the augmented system settling time, then the states from the first may be used to reestimate the dynamics of the second. The estimation problem is now identical to that of the literature [8,13,18] since the accurate states are now available for the estimation process

The observer system is released with its initial conditions set to the best guess of the states. After one settling time, accurate states will be available for estimation of the dynamics of the system estimate using the method of Gates [13]. The system estimate is needed to use as a trial horse against the observed system behavior. These systems may be successively stepped to the desired convergence or to follow a slowly time-varying system or a weakly nonlinear system.

Time-varying, linear systems become more interesting and more complex. Now the dynamics of the estimator must be rapid enough to track the system under expected variations to fulfill the response matching criterion. The state reconstruction dynamics are chosen to be faster than the estimator within considerations of physical restrictions and overall stability. For nonlinear systems, one may consider first those that possess only small signal linearities or those that are weakly nonlinear. A well-known artifice of control analysis is to treat weakly nonlinear systems as time-varying linear systems. This approximation is not without risk, since an unstable system may appear to be stable in this type of analysis and vice versá. If the time-varying approximation is permitted, the technique may be applied as with the time-varying case with the additional restriction of defining a region for which the response remains sufficiently weakly nonlinear. The example of Chapters IV and V is of this type.

Stability considerations for these processes are concerned with three distinct elements. The stability of the plant estimate is of primary importance. Parameter variations in the estimation process may move marginally stable eigenvalues into the right half plane. If this occurs special tuning techniques are required to achieve the desired accuracy due to the observer [16] behavior. The second element is the augmented system eigenvalues of $A + KC^T$. In practice this is seldom a difficult task because the elements of K are quite large in the interest of rapid system response. The variations in parameter estimates are normally small, in relation to the elements of K; thus, augmented system eigenvalues tend to be insensitive. The last consideration is the stability of the estimation algorithm. Many techniques exist [18] to assure the stability of this process.

System Diagrams and Equation Development

The equations and block diagrams will now be developed to give form to the method. Since most implementation schemes are digital in nature, the equations and algorithm will be developed in discrete form.

The implementation has, as its objectives, the recovery of the unmeasured states and the estimation of the system parameters. The unmeasured and measured states are direct outputs of the observer previously described. These states are used directly in the estimation process developed by Gates [13]. Figure 3-1 is a block diagram of the system estimation process. The plant dynamics are differenced with the dynamics of the reference model to form an error which is used to determine the difference between the reference model and the plant. The reference model is updated with the calculated difference, scaled by an appropriate gain. This gain is chosen for the stability of the estimation process. The process is repeated until the desired convergence is achieved.

Figure 3-2 portrays the combined state reconstruction and parameter estimation process. The plant, the observer, and the reference model are driven by the input U. The observer is also driven by the measured output of the plant. An error is formed from the difference of the observer and the reference model. Due to the behavior of the robust observer, the plant estimate $\hat{x}$ will follow the plant x. The observer and the reference model use the same estimate of the plant. The previously determined error will be used to calculate the difference between the plant and the reference model. The algorithm performs this function. The reference model and the observer are updated by use of the calculated difference and again scaled by an appropriate gain chosen for estimation stability. The process is repeated until a desired degree of convergence is achieved.

The equations will now be developed considering known external inputs. The plant is assumed governed by

$$x(k + 1) = G(T)\, x(k) + H(T)\, U(k) \qquad (3\text{-}1)$$

where

$$G(T) = e^{AT} \tag{3-2}$$

and

$$H(T) = \int_{kT}^{(k+1)T} e^{A\tau} B \, d\tau \tag{3-3}$$

with measurements

$$y(k) = C^T x(k) \quad . \tag{3-4}$$

A is the system Jacobian, U is the known input and B is the control distribution matrix. The observer is described by

$$\hat{x}(k+1) = G_1'(T)\,\hat{x}(k) + H_1(T)\,U(k) + L_1'(T)\,y(k) \tag{3-5}$$

where

$$G_1'(T) = e^{(D+KC^T)T} \tag{3-6}$$

$$L(T) = \int_{kT}^{(k+1)T} e^{(D+KC^T)\tau} B \, d\tau \tag{3-7}$$

and

$$L_1' = \int_{kT}^{(k+1)T} e^{(D+KC^T)\tau} K \, d\tau \tag{3-8}$$

D is the estimate of the system Jacobian, and K is the observer gain matrix.

The reference model is expressed as

$$x_1(k + 1) = G_1(T)\, x_1(k) + H_1(T)\, U(k) \tag{3-9}$$

where

$$G_1(T) = e^{DT} \tag{3-10}$$

and

$$H_1(T) = \int_{kT}^{(k+1)T} e^{D\tau} B \, d\tau \tag{3-11}$$

Now, writing state in terms of the system estimate

$$x(k + 1) = G(T)\, x(k) + \delta G(T)\, x(k) + H_1(T)\, U(k) + \delta H(T)\, U(k) \quad . \tag{3-12}$$

The error between the estimate and the state may be expressed as

$$e(k + 1) = G_1(T)\, e(k) + \delta G(T)\, x(k) + \delta H(T)\, U(k) \tag{3-13}$$

The following matrices are formed

$$\begin{aligned} x0 &= (x(0), \ldots, x(-n - p + 1)) \\ U0 &= (U(0), \ldots, U(-n - p + 1)) \\ e0 &= (e(0), \ldots, e(-n - p + 1)) \\ e1 &= (e(1), \ldots, e(-n - p + 2)) \end{aligned} \tag{3-14}$$

where each column is the vector associated with the enumerated time point. Eq. (3-13) can be expressed as a matrix equation that has

$$(\delta G(T):\delta H(k)) = (eI - G_1(T)\ e0) \begin{bmatrix} x_0 \\ \cdot\cdot \\ U_0 \end{bmatrix}^{-1} \tag{3-15}$$

as a solution if the inverse of the augmented state and control matrix exists.

The system description is now updated as

$$G_1(T) = G_1(T) + \delta G(T) \tag{3-16}$$

$$H_1(T) = H_1(T) + \delta H(T) \quad , \tag{3-17}$$

and

$$G_1'(T) = G_1'(T) + \delta G(T) \quad . \tag{3-18}$$

Eq. (3-18) is not exactly correct but, due to the magnitude of elements of K, yields satisfactory results. Several alternatives exist to precisely calculate $G_1'(T)$. One method is to recover δD from $\delta G(T)$ and recalculate $G_1'(T)$ from Eq. (3-6). Another method is to calculate as a continuous system using the numerical integration to discretize the system. The calculation, in this case, yields δD. A variation of the last method would be to use the δD to calculate the discrete parameters of Eqs. (3-6), (3-7), (3-8), (3-10), and (3-11). Precise knowledge of $G_1'(T)$ is not required due to the nature of the robust observer. A topic for future study is the development of a recursive solution to the previously described estimation.

A Second Order Example

To demonstrate the procedure the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3-19}$$

with measurement

$$y = (0 \quad 1)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_2 \tag{3-20}$$

Discretizing this system results in difference equations of the form

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ -2e^{-T} + 2e^{-2T} & -e^{-T} + 2e^{-2T} \end{bmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \tag{3-21}$$

where T is the sampling period.

First the observer dynamics will be developed. The error between the state and the state estimate is dynamically determined by the eigenvalues of $A + KC^T$, or

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} + \begin{bmatrix} 0 & K_1 \\ 0 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & K_1 + 1 \\ -2 & K_2 - 3 \end{bmatrix} \tag{3-22}$$

which has a characteristic equation of

$$\lambda^2 + (3 - K_2)\lambda + (2K_1 + 2) = 0 \quad . \tag{3-23}$$

Hence the values for

$$\lambda = \frac{K_2 - 3 \pm \sqrt{1 + K_2^2 - 6K_2 - 8K_1}}{2} \tag{3-24}$$

are the robust observer eigenvalues and may be placed almost arbitrarily. Roots may both be real as $K_2 = -2$, $K_1 = 2$ which has roots at -2 and -3. Figure 3-3 shows the response of the unmeasured state and state estimate with a perfect model. The settling time is almost 3 sec. Figure 3-4 considers the response for a reasonable estimate of the state. Now the settling time is almost 4 sec. These dynamics are too slow for application of the estimation technique. If $K_1 = 24$, $K_2 = -12$ which has roots at -5 and -10 then the dynamics are much more suitable to the application of the estimation process. Figure 3-5 presents the response for these observer dynamics and a perfect model. Notice the settling time is of the order of 0.75 sec. Figure 3-6 shows the response for a reasonable estimate of the system dynamics. Again the settling time is less than 1 sec.

Figure 3-7 shows the remarkable property of robust observers to converge to the state even though there are errors in the plant estimate. The settling time is on the order of 3 sec, and this can be improved by adjusting the observer. The observers are all processing data from the plant

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x \qquad (3\text{-}25)$$

while the reasonable estimate was

$$\dot{\hat{x}} = \begin{pmatrix} -0.2 & 0.9 \\ -2.2 & -3.1 \end{pmatrix} \hat{x} - KC^T(x - \hat{x}) \qquad (3\text{-}26)$$

The gross model error is characterized as

$$\dot{\hat{x}} = \begin{pmatrix} -0.5 & 2 \\ -3 & -6 \end{pmatrix} \hat{x} - KC^T(x - \hat{x}) \qquad (3\text{-}27)$$

which is considerably at variance with Eq. (3-25).

The parameter estimation is delayed until reasonable convergence of the observer is achieved. The estimation is then initiated with the error between the trial system and the observer defined to be zero. Figure 3-8 presents a plot of time versus parameter estimate showing the estimation convergence.

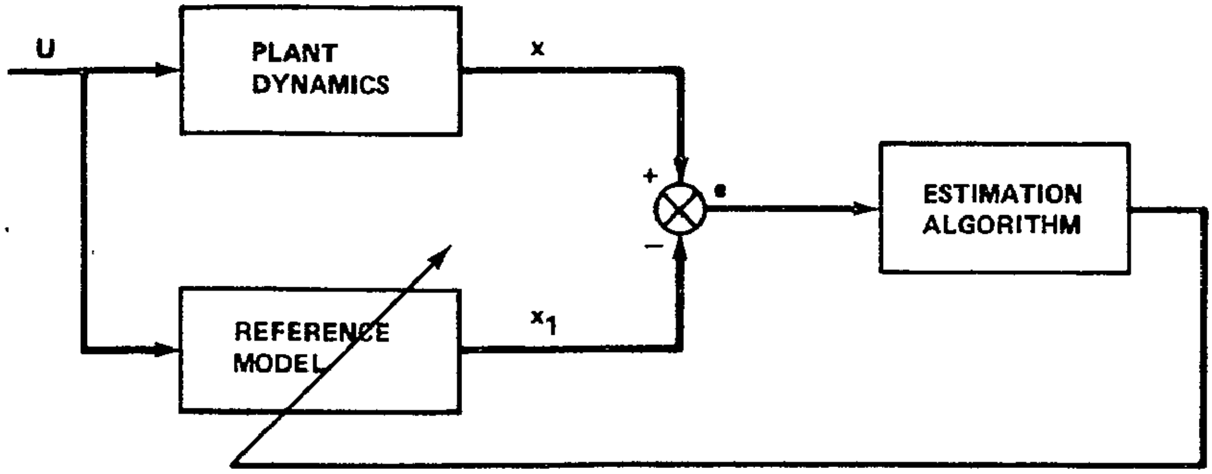

Figure 3-1. Block diagram of the estimation process.

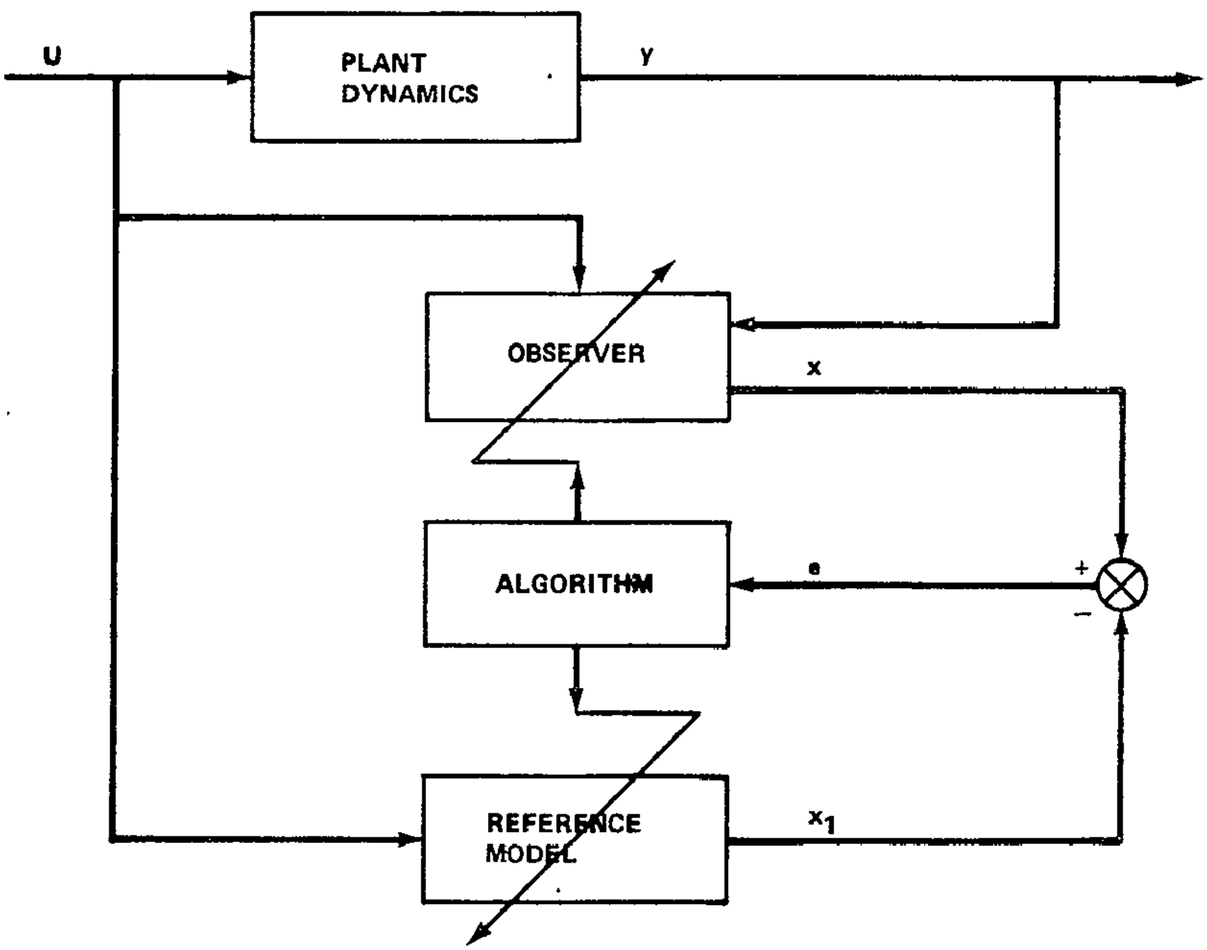

Figure 3-2. Block diagram of the combined reconstruction and estimation.

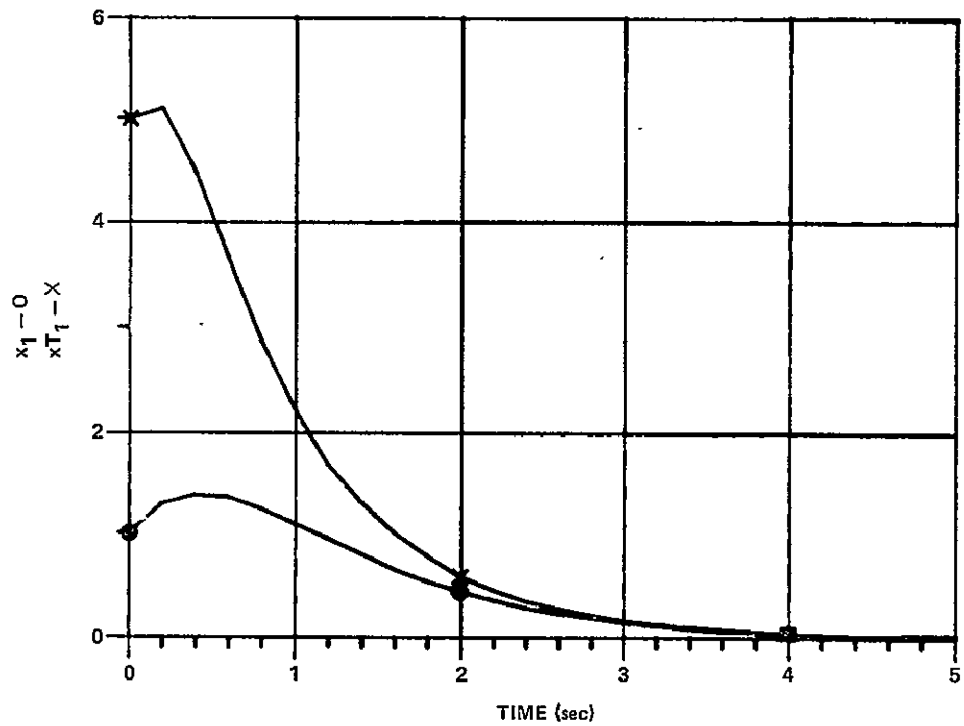

Figure 3-3. Observer with perfect model, slow dynamics (K_1 = -2, K_2 = 2).

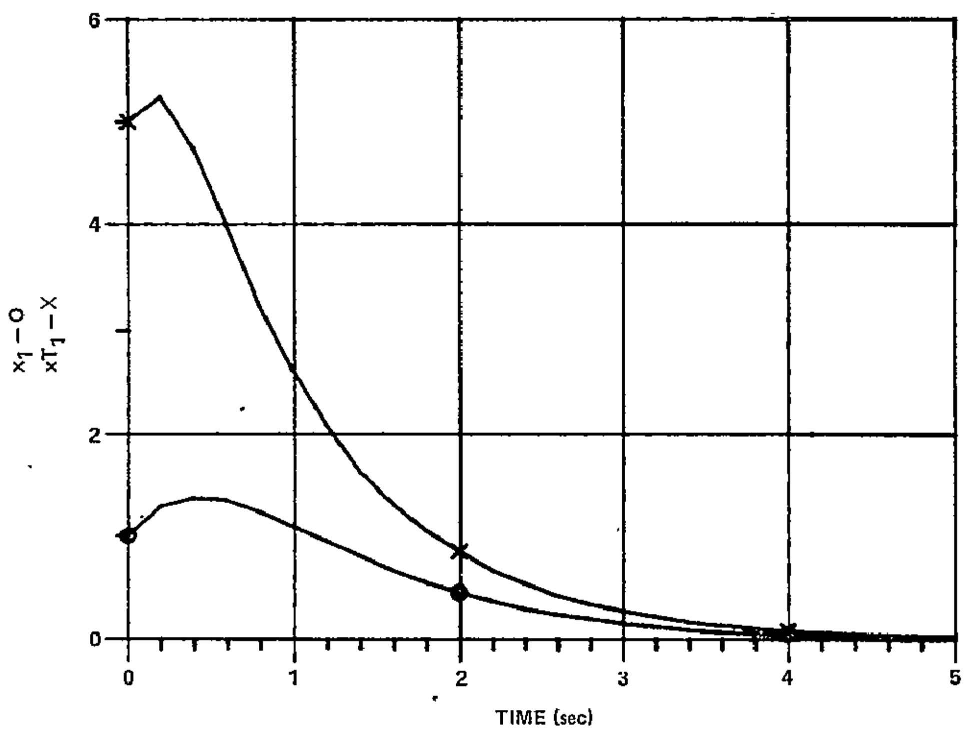

Figure 3-4. Observer with reasonable estimate, slow dynamics (K_1 = -2, K_2 = 2).

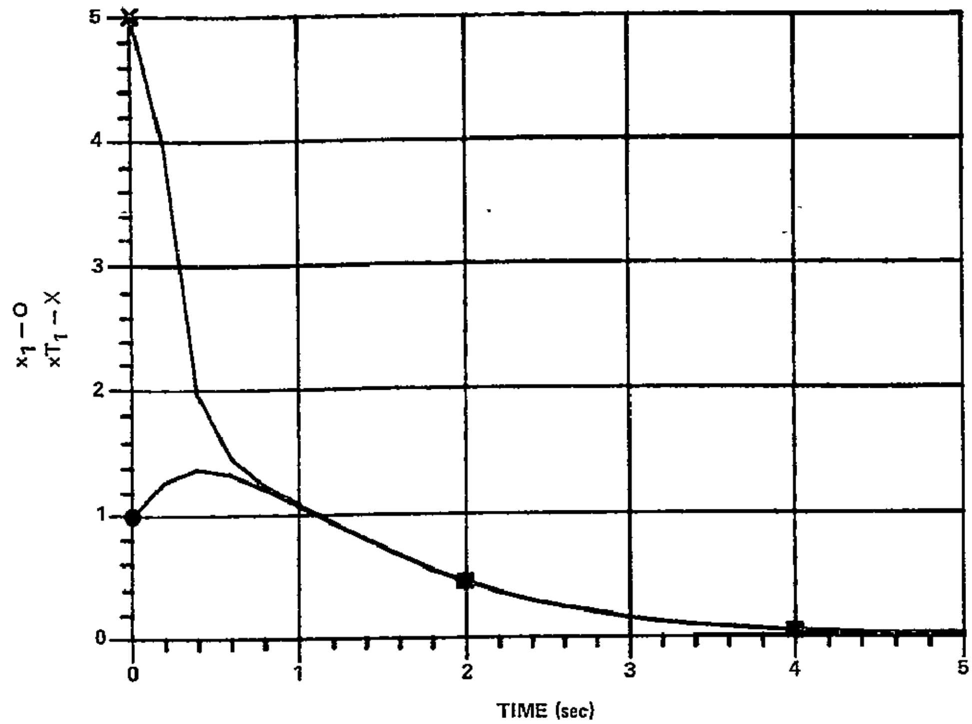

Figure 3-5. Observer for perfect model.

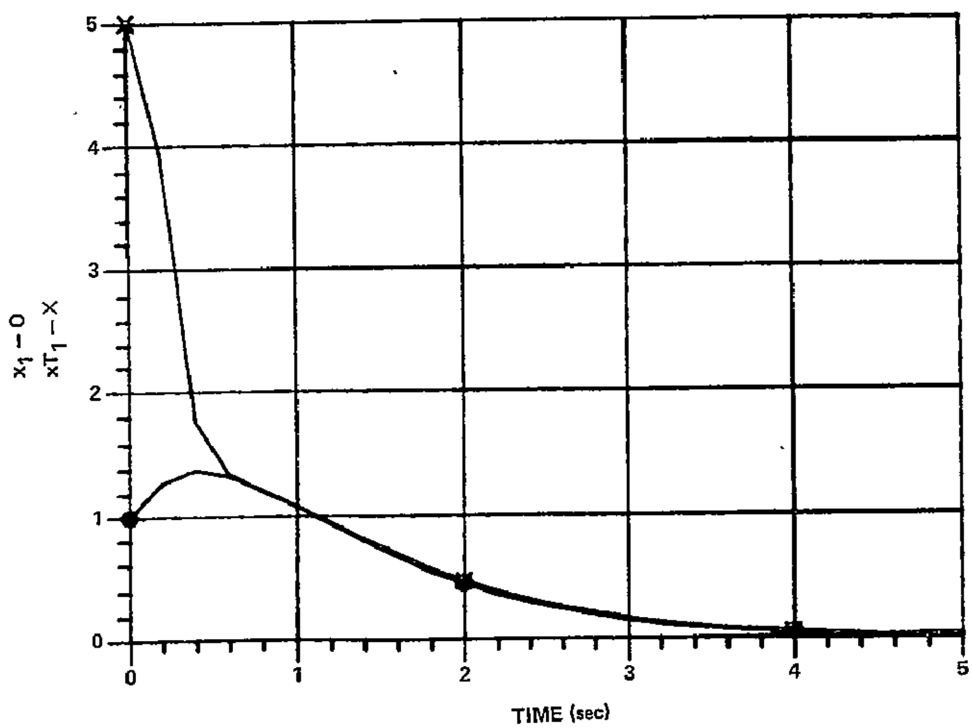

Figure 3-6. Observer with reasonable estimate of model.

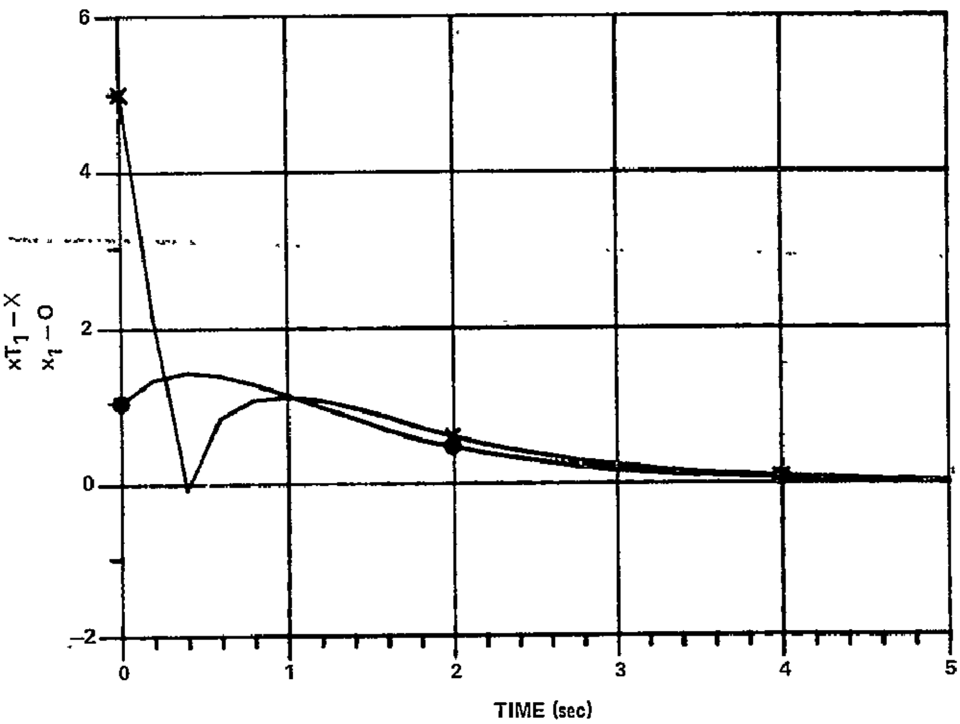

Figure 3-7. Observer with gross model error.

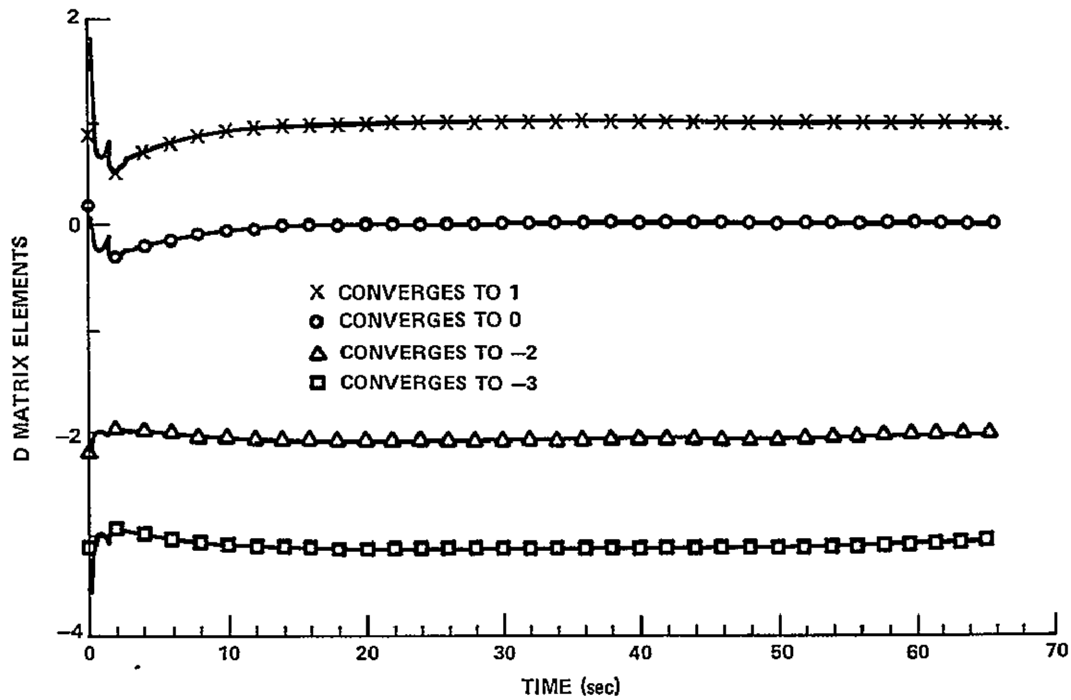

Figure 3-8. Observer (24, -12) convergence of D to the A matrix.

CHAPTER IV
CTL-V TESTING ANALYSIS

All large space vehicles possess a longitudinal dynamic coupling of structure and propulsion predictably called "POGO." The most benign stage to date is the Saturn IB stage which has nine tanks in a bundle and eight engines providing a maximum of statistical interaction, which results in overall system damping. The Space Shuttle, due to its large single LOX feedline, is expected to be susceptible to POGO. Due to this concern, an accumulator is being designed for the Space Shuttle main engine as a decoupling and suppressive device. A primary qualification test for this device is the CTL-V Test series at the Rockwell International Rocketdyne Division, Santa Suzanna Test Facility. These tests will provide assurance as to the dynamic representation of the low pressure oxidizer pump and the effect and effectivity of the accumulator.

The dynamic head rise characteristics of the low pressure oxidizer pump are nonlinear and not precisely known. They are modeled as nonlinear differential equations whose coefficients are empirically determined parameters. The accumulator characteristics are also ill defined because of the difficulty of obtaining good test results and isolation of the higher frequency effects of the accumulator. The advantages of applying the previously developed techniques are that by matching the time response the frequency response is likewise adjusted. That is, the linearization of the appropriate time response provides a frequency domain representation of the dynamic phenomena. This should be a "best" linear representation at that condition because the estimate is being forced to behave in a fashion similar to the actual system. The method is developed and modified to provide a neighborhood of operation of the low pressure oxidizer pump and of the accumulator.

The pump modeling is defined by the Rocketdyne publication RL00001 [19] defining the Space Shuttle main engine-engine balance and dynamic model. The facility is modeled in a similar fashion. Since the lines and pumps will be essentially chilled to a steady state during a given test, the assumption of incompressibility and thermal steady state is valid. Tests have shown that there is energy trade between temperature and pressure but that these are small effects. The specific objectives of the analysis will be to better define the head rise dynamics of the pump and the parameters of the accumulator. The accumulator parameters are characterized by electromechanical analogy. These parameters consist of a compliance, an inertance, and an equivalent resistance. The reason for an equivalent resistance will be apparent in the equation development. The results of these tests and analyses will be used in the overall Space Shuttle POGO stability analysis to better define system stability before first flight.

CTL-V Equation Development

The system to be tested in CTL-V is that of Figure 4-1. The pump speed S_{OI} may be assumed constant since the pump is being driven by an extremely high inertia electric motor. The constant pump speed allows analysis of the basic head rise characteristics of the pump uncoupled from available drive torque and torque required which

couple back into flow and pump speed. P_T is the pressure at the feedline inlet. The flow in the feedline DW_{FL} may be represented as

$$DW_{FL} = \frac{1}{L_L} \int_0^t \left[(P_T - P_{OS}) - R_L \, DW_{FL}^2 \right] d\tau \quad . \tag{4-1}$$

The "bubble" on the pump has pressure P_{OS}.

$$P_{OS} = \frac{1}{C_B} \int_0^t (DW_{FL} - DW_{OS}) \, d\tau \quad . \tag{4-2}$$

These two elements combine to simulate the 2.5 Hz first resonance of the ozidizer feedline. All damping arises from the resistive term of the feedline flow Eq. (4-2).

Next is the low pressure oxidizer pump. The pump is assumed to have a dynamic gain of one. Mass continuity dictates that the pump be gain one at zero frequency. The pump is characterized simply as a nonlinear head rise device. The pump discharge pressure is

$$P_{OD1} = P_{OS} + H' \quad . \tag{4-3}$$

The head rise H' will be defined by use of a dimensionless parameter

$$\Phi_{OP1} = \frac{A_1}{S_{O1}} \, DW_{OS} \quad . \tag{4-3a}$$

The head rise itself is given by

$$H' = B_1 S_{O1}^2 \, \Gamma_{POP1}(\Phi_{OP1}) \tag{4-3b}$$

where $\Gamma_{POP1}(\Phi_{OP1})$ is determined from the empirical curve of Figure 4-2. The flow below the pump is the same as the flow existing in the bubble and entering the pump and is defined as

$$DW_{OS} = \frac{1}{L_D} \int_0^t \left[(P_{OD1} - P_{OI2}) - R_D (DW_{OS})^2 \right] d\tau \quad . \tag{4-4}$$

L_D is the inertance of the fluid in the duct and the pump. R_D is a lumped resistance coefficient combining effects of duct and pump. The pressure upstream of the accumulator is dependent on the compliance, C_D, of the duct itself, and has the form

$$P_{OI2} = \frac{1}{C_D} \int_0^t (DW_{OS} - DW_{OP2} - DW_A)\, d\tau \quad . \tag{4-5}$$

The accumulator is modeled analogously with a pressure change through a compliance and a flow change due to a resistance and delta pressure. The inertance in this case is the mass of that fluid trapped in the standpipe leading to the accumulator. The compliance is a lumped compliance consisting of flexure of the housing and the compressibility of the gas in the accumulator. The gas to be used on the Space Shuttle is GOX in contrast to helium principally used in the past. The GOX is supplied from the tank pressurization heat exchanger and maintains a constant level in the accumulator by use of an overflow port from which GOX is vented back into the feedline above the low pressure pump. That flow is not considered in this analysis. The gas-liquid interface is maintained by four layers of 3/8 in. Teflon balls which provide internal slosh suppression and help prevent the gas bubble from collapsing in the liquid. The pressure in the accumulator is modeled as

$$P_A = \frac{1}{C_A} \int_0^t DW_A \, d\tau \quad , \tag{4-6}$$

while the accumulator flow can be represented as

$$DW_A = \frac{1}{L_A} \int_0^t \left[(P_{OI2} - P_A) - R_A (DW_A)^2 \right] d\tau \qquad (4\text{-}7)$$

where C_A, L_A, and R_A are as previously defined.

P_{OP2} is, in effect, an output of the system and, because of the orifice, will remain constant once the test conditions are established. With one exception, the equation definition is now complete. The exception is that to represent P_{OI2}, the flow downstream of the accumulator must be modeled. This is a small piece of fluid and results in a high frequency root. L_N represents the inertance of that element and R_N is its resistance. The flow in this section can then be expressed as

$$DW_{OP2} = \frac{1}{L_N} \int_0^t \left[(P_{OI2} - P_{OP2}) - R_N (DW_{OP2})^2 \right] d\tau \quad . \qquad (4\text{-}8)$$

This completes the equation development required to analyze the test.

These equations may be rewritten as a set of nonlinear differential equations:

$$D\dot{W}_{FL} = \frac{1}{L_L} \cdot P_T - \frac{1}{L_L} P_{OS} - \frac{R_L}{L_L} DW_{FL}^2 \qquad (4\text{-}9)$$

$$\dot{P}_{OS1} = \frac{1}{C_B} DW_{FL} - \frac{1}{C_B} DW_{OS} \qquad (4\text{-}10)$$

$$\dot{P}_{OD1} = \frac{1}{C_B} DW_{FL} - \frac{1}{C_B} DW_{OS} - A_1 B_1 S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{R_D}{L_D} DW_{OS}^2$$

$$+ A_1 B_1 S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{1}{L_D} P_{OD1} - A_1 B_1 S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{1}{L_D} P_{OI2}$$

(4-11)

$$\dot{DW}_{OS} = \frac{1}{L_D} P_{OD1} - \frac{1}{L_D} P_{OI2} - \frac{R_D}{L_D} DW_{OS}^2 \tag{4-12}$$

$$\dot{P}_{OI2} = \frac{1}{C_D} DW_{OS} - \frac{1}{C_D} DW_{OP2} - \frac{1}{C_D} DW_A \tag{4-13}$$

$$\dot{P}_A = \frac{1}{C_A} DW_A \tag{4-14}$$

$$\dot{DW}_A = \frac{1}{L_A} P_{OI2} - \frac{1}{L_A} P_A - \frac{R_A}{L_A} DW_A^2 \tag{4-15}$$

$$\dot{DW}_{OP2} = \frac{1}{L_N} P_{OI2} - \frac{1}{L_N} P_{OP2} - \frac{R_N}{L_N} DW_{OP2}^2 \quad . \tag{4-16}$$

Linearized Analysis of CTL-V

These equations are in turn linearized to obtain the following set of linear differential equations:

$$\Delta\dot{F}_{FL} = \frac{1}{L_L} \Delta P_T - \frac{1}{L_L} \Delta P_{OS} - 2 \frac{R_L}{L_L} DW_{FL} \Delta F_{FL} \quad . \tag{4-17}$$

The term $2R_L DW_{FL}$ is normally thought of as being an equivalent damping resistance for an element.

$$\Delta\dot{P}_{OS} = \frac{1}{C_B} \Delta F_{FL} - \frac{1}{C_B} \Delta F_{OS} \tag{4-18}$$

$$\Delta\dot{P}_{OD1} = \frac{1}{C_B} \Delta F_{FL} - D_1 \Delta F_{OS} + D_2 \Delta P_{OD1} - D_2 \Delta P_{OI2} \tag{4-19}$$

where

$$D'_1 = \frac{1}{C_B} - 2A_1B_1S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{R_D}{L_D} DW_{OS}$$

$$D'_2 = A_1B_1S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{1}{L_D} \quad .$$

Completing the equations,

$$\Delta \dot{F}_{OS} = \frac{1}{L_D} \Delta P_{OD1} - \frac{1}{L_D} \Delta P_{OI2} - \frac{2R_D DW_{OS}}{L_D} \Delta F_{OS} \tag{4-20}$$

$$\Delta \dot{P}_{OI2} = \frac{1}{C_D} \Delta F_{OS} - \frac{1}{C_D} \Delta F_{OP2} - \frac{1}{C_D} \Delta F_A \tag{4-21}$$

$$\Delta \dot{P}_A = \frac{1}{C_A} \Delta F_A \tag{4-22}$$

$$\Delta \dot{F}_A = \frac{1}{L_A} \Delta P_{OI2} - \frac{1}{L_A} \Delta P_A - \frac{2R_A}{L_A} DW_A \Delta F_A \tag{4-23}$$

$$\Delta \dot{F}_{OP2} = \frac{1}{L_N} \Delta P_{OI2} - \frac{2R_N}{L_N} DW_{OP2} \Delta F_{OP2} \quad . \tag{4-24}$$

Notice that P_{OP2} has dropped out of the linear representation because it is assumed constant due to the orifice. At the beginning of the chapter, the equivalent resistance of the accumulator was discussed. The resistance is equivalent because, to a first-order approximation, it is zero. In Eq. (4-23), the linear term of resistance is $2R_A/L_A\ DW_A$. In the steady state DW_A is zero. Therefore, to first order the resistance effects of the accumulator are indeed zero. However, the resistive effects are important to the analysis so a small nonzero term will be forced in the analysis to assess its effect.

These equations can now be expressed in state variable form as

$$\dot{x} = Ax + BU \quad ; \quad \text{where } x = \begin{pmatrix} \Delta P_{OS} \\ \Delta F_{FL} \\ \Delta P_{OD1} \\ \Delta F_{OS} \\ \Delta P_{OI2} \\ \Delta F_{OP2} \\ \Delta P_{A} \\ \Delta F_{A} \end{pmatrix}$$

where

$$A = \begin{pmatrix} 0 & \frac{1}{C_B} & 0 & -\frac{1}{C_B} & 0 & 0 & 0 & 0 \\ -\frac{1}{L_L} & \left(-\frac{2R_L}{L_L} DW_{IL}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_B} & D_2 & -D_1 & -D_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_D} & \left(-\frac{2R_D}{L_D} DW_{OS}\right) & -\frac{1}{L_D} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_D} & 0 & -\frac{1}{C_D} & 0 & -\frac{1}{C_D} \\ 0 & 0 & 0 & 0 & \frac{1}{L_N} & \left(-\frac{2R_N}{L_N} DW_{OP}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_A} \\ 0 & 0 & 0 & 0 & \frac{1}{L_A} & 0 & -\frac{1}{L_A} & -\frac{R_A}{L_A} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$U = P_T$$

In practice, flow is an unmeasurable quantity due to instrumentation difficulties. For this reason the pressures are all that will be measured. The advantages of the scheme presented over most identification schemes are now obvious. Fully half of the state vector is not available for measurement and will be recovered with an observer designed in a fashion described previously.

The measurement vector now becomes

$$\begin{pmatrix} P_{OS} \\ P_{OD1} \\ P_{OI2} \\ P_A \end{pmatrix} = C^T \begin{pmatrix} P_{OS} \\ F_{FL} \\ P_{OD1} \\ F_{OS} \\ P_{OI2} \\ F_{OP2} \\ P_A \\ F_A \end{pmatrix} \tag{4-26}$$

These equations will be discretized using Fourth Order Runge-Kutta Integration, with iteration time sufficiently fast to assure reasonable accuracy. This provides satisfactory precision without cumbersome implementation.

The test series will be operated in two ways, with and without the accumulator. The equations have been arranged to allow partitioning in this manner. Using data that reflect the rated power level test, the eigenvalues of the system without accumulator are as follows:

Real	Imaginary
-32.45	-224.4
-32.45	224.4
-59.1	71.7
-59.1	-71.7
-5.3	0
-0.01	0

Notice that the line frequency has dropped from 2.5 to 1.2 Hz. The pump and duct combine with critically damped roots, and the orifice segment has a 36 Hz resonance.

Adding the accumulator, the eigenvalues become:

Real	Imaginary
-81.8	-423.1
-81.8	423.1
-72.7	77.9
-72.7	-77.9
-37.5	48.7
-37.5	-48.7
-5.35	0
-0.01	0

The line frequency is essentially unchanged as is the pump and upper duct. The major change is in the lower duct where the duct couples with the accumulator giving the 15 Hz resonance and the accumulator couples with the flow to the orifice which has a 69 Hz resonance.

It remains to locate the eigenvalues of the observer and implement the identification technique.

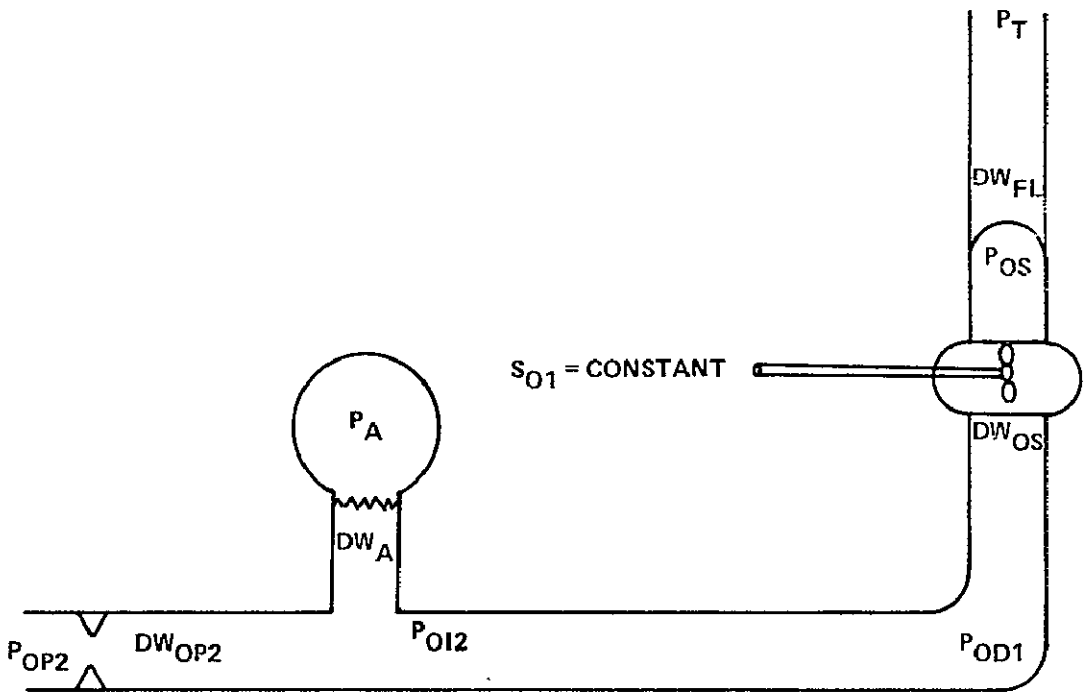

Figure 4-1. CTL-V test.

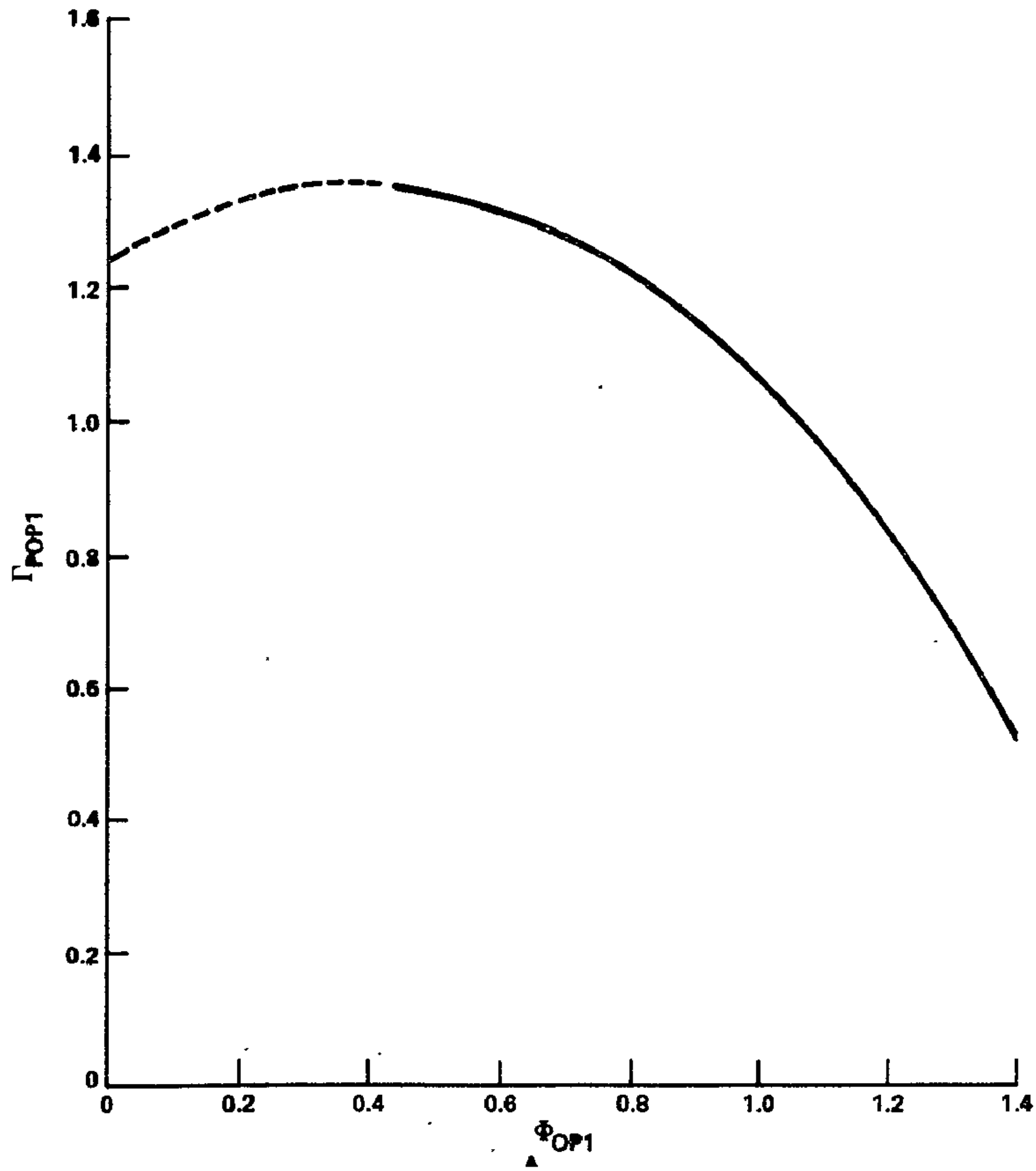

Figure 4-2. Low pressure oxidizer pump pressure rise characteristics.

CHAPTER V
CTL-V RESULTS

The problem of higher-order observer design, analysis, and performance will be discussed first, then the actual design will be developed. Results will include those of the CTL-V facility with and without the POGO suppressive accumulator. The principal feature is the demonstration that linear observers can be used in a state-parameter estimation process if sufficient care is taken in the design to insure plant-observer dynamic decoupling. Techniques for desensitizing observer design and the application of parameter estimating state reconstruction will be examined. Sample rate selection and numerical difficulties will be addressed. Finally the curious phenomena of multiple equilibriae for fluidic systems of the CTL-V type will be analyzed.

CTL-V Observer Design

The design of observers for higher-order systems is a topic in itself. The simplest problem is the single input system. If the subject system is observable, there are available n times m parameters, of the gain matrix K, to place the n eigenvalues of the augmented observer system. If there are multiple inputs, the system may be recast as a set of single input systems and treated individually as single input systems. However, while systems designed in this fashion have the desired eigenvalues, the dynamics of the augmented system can be most undesirable because of the location of the system zeros. Undesirable energy trade takes place between the various component single-input systems. The additional degrees of freedom, in the matrix K, may be used to achieve a more desirable overall dynamic response. The term "better dynamic response" must now be quantified. For the purpose of this research, better dynamic response means critically damped with a reasonably fast settling time, while in general the term is dependent on the application and the desires of the designer.

A critically damped response is desired to eliminate or reduce coupling between the plant and the observer. A further precaution is to design the augmented system eigenvalues sufficiently larger than those of the plant. This permits rapid reconstruction of the states and reduces the propensity of the observer dynamics to couple with the plant dynamics. The requirement to critically damp the observer eigenvalues means that some of the desired analysis flexibility has been lost and that the technique is becoming more system dependent. The critical damping also affects the settling time which is another design parameter. The settling time determines how often new estimates of the plant parameters can be determined. Due to the nature of observers, only the directly measured states are known before the reconstruction process. The unmeasured states become available only after the system settling time has passed. This time can also be affected by the size of the error in the initial estimates of the unmeasured states. A settling time is required after each recalculation of the system parameters because the new estimate represents a system discontinuity when it is used in the reconstruction process.

A consideration that would be of low interest to most applications, but is of secondary interest in this application, is the augmented system eigenvalues sensitivity to parameter variations. This interest is of two parts: first the sensitivity of the observer dynamically to deviations in the plant estimate and, second, the stability of the estimated systems eigenvalues due to estimation errors. Due to the nature of the robust observer, the dynamic behavior may be degraded as variations become large. But the system will converge for very large variations in the parameter estimates, as was demonstrated in Chapter III. In fact, the whole concept of parameter estimating state reconstruction is based on that property. But gross excursions can cause divergence of the reconstructor from the plant, and some designs are more or less sensitive to parameter variations. A design procedure then is to verify a low sensitivity to parameter variations.

The second part of the problem is the closed-loop stability of the estimation system. The augmented system may be stable but may perform inadequately for the purposes of this research. If the parameter recalculation causes the observer plant to have unstable eigenvalues, then quite obviously the system will have inappropriate dynamics. Therefore, another design procedure is to analyze the sensitivity of the plant eigenvalues to parameter variations and the overall degree of stability of the object plant. A marginally stable or unstable system is undesirable for analysis by this technique because of observer problems addressed in Chapter II, and the overall system sensitivity. The observer problem is that if the plant is unstable, then the plant must be precisely known and represented in the observer to achieve observer convergence. However, an artifice is available to handle these kinds of difficulties, that is, to synthesize a feedback control that stabilizes or desensitizes the objectionable eigenvalues. This is a straightforward classical technique that provides a system with characteristics that permit analysis by the technique of this research, the control being included in the model structure of the new observer of the altered system.

The eigenvalue placement for observers, as has been noted, is generally overdetermined. There are a variety of ways to choose the elements of the gain matrix K. The particular approach for this application was selected because of the manner in which the elements of KC^T enter the augmented system matrix $A + KC^T$. By choosing K in the form

$$KC^T = \begin{pmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 \\ K_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{32} & 0 & 0 & 0 \\ 0 & 0 & K_{42} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{53} & 0 \\ 0 & 0 & 0 & 0 & K_{63} & 0 \end{pmatrix} \qquad (5\text{-}1)$$

for the sixth order case, and

$$KC^T = \begin{pmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{32} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{42} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{53} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{63} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{74} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{84} & 0 \end{pmatrix} \tag{5-2}$$

for the eight order case, the augmented system eigenvalues can be less interactively chosen. The system is naturally partitioned to encourage this type of gain selection. There is some interaction, but eigenvalue selection is more independent than if some more-coupled scheme were used. Values for K and the associated augmented system eigenvalues are shown in Table 5-1. Sensitivity results for sixth order observers are shown in Table 5-2 and for eighth order observers in Table 5-3. The numbers of the K matrix are in units commensurate with the elements of the D or A matrix so that $A + KC^T$ has meaning. If the selected elements of K are extremely large, errors in the parameter estimates have little effect on the observer dynamics; however, due to the high gain, the observer system becomes very sensitive to noise. If the elements of K are small the observer becomes more sensitive to parameter estimate errors and the observer response becomes sluggish. These considerations enter the observer system design process.

Physical Interpretation of the Model

The desired output is not simply the linear model, or the matrix D in the calculations. D must be interpreted to deduce the parameters of interest, namely the compliances, inertances, and resistances of the CTL-V facility but, most importantly, the slope of the pump curve. The compliance of the bubble on the pump may be determined as

$$C_B = \frac{1}{D_{12}} \tag{5-3}$$

while the feedline inertance is

$$L_L = \frac{1}{D_{21}} \quad . \tag{5-4}$$

TABLE 5-1. SIXTH AND EIGHTH ORDER OBSERVER

	Sixth Order		Eighth Order	
	Value	Eigenvalues[a] (All Real)	Value	Eigenvalues[a] (All Real)
K_{11}	1200.0	-100.0	1400.0	-96.3
K_{21}	1998.0	-219.3	1998.0	-138.3
K_{32}	1477.0	-354.3	1277.0	-272.3
K_{42}	0.0	-858.3	0.0	-544.3
K_{53}	1500.0	-1328.1	1500.0	-859.7
K_{63}	-1366.0	-1496.4	-1366.0	-1127.6
K_{74}	—	—	1000.0	-1212.2
K_{84}	—	—	1500.0	-1305.8

a. Eigenvalues have no order relationship to values.

The line resistance is a little more difficult to recover but may be determined as

$$R_L = \frac{-D_{22} L_L}{2\dot{W}_L} \quad . \tag{5-5}$$

Next the inertance of the duct may be calculated as

$$L_D = \frac{1}{D_{43}} \quad . \tag{5-6}$$

TABLE 5-2. SIXTH ORDER SENSITIVITY

	Value	Eigenvalue[a]		Value	Eigenvalue[a]		Value	Eigenvalue[a]	
		Real	Imaginary		Real	Imaginary		Real	Imaginary
K_{11}	1600.0	-97.2	0.0	1600.0	-168.4	87.3	1600.0	-184.9	0.0
K_{21}	1998.0	-207.6	0.0	2498.0	-168.4	-87.3	2498.0	-185.1	59.1
K_{32}	1477.0	-238.8	0.0	1777.0	-198.0	0.0	2477.0	-185.1	-59.1
K_{42}	0.0	-1325.9	0.0	0.0	-1337.1	0.0	623.0	-1339.1	5.6
K_{53}	1500.0	-1378.9	0.0	1500.0	-1370.0	0.0	1500.0	-1339.1	-5.6
K_{63}	-1366.0	-1508.2	0.0	-1366.0	-1814.7	0.0	-1366.0	-2523.1	0.0

a. Eigenvalues have no order relationship to values.

TABLE 5-3. EIGHTH ORDER SENSITIVITY

	Eigenvalues[a]			Eigenvalues[a]			Eigenvalues[a]		
	Value	Real	Imaginary	Value	Real	Imaginary	Value	Real	Imaginary
K_{11}	1000.0	-98.6	0.0	1000.0	-94.6	0.0	1000.0	-95.6	0.0
K_{21}	1998.0	-139.5	0.0	1998.0	-146.0	0.0	1998.0	-159.3	0.0
K_{32}	1277.0	-540.6	0.0	727.0	-493.3	-236.4	727.0	-492.8	-236.9
K_{42}	0.0	-506.6	-212.4	0.0	-493.3	236.4	0.0	-492.8	236.9
K_{53}	1500.0	-506.6	212.4	1000.0	-559.8	-393.0	1000.0	-555.5	-443.8
K_{63}	-1366.0	-860.4	0.0	-1266.0	-559.8	393.0	-1666.0	-555.5	443.8
K_{74}	1000.0	-1209.9	0.0	875.0	-778.8	0.0	875.0	-778.9	0.0
K_{84}	1500.0	-1294.4	0.0	800.0	-855.8	0.0	800.0	-850.9	0.0

a. Eigenvalues have no order relationship to value.

The slope of the pump curve may now be evaluated as

$$\frac{\partial \Gamma}{\partial \Phi} = \frac{D_{33} L_D}{A_1 B_1 S_{O1}} \tag{5-7}$$

where A_1B_1 is a constant and S_{O1} is the pump speed. The compliance of the duct is simply

$$C_D = \frac{1}{D_{54}} \quad . \tag{5-8}$$

The resistance of the duct is

$$R_D = \frac{-D_{44} L_D}{2\dot{W}_D} \quad . \tag{5-9}$$

The inertance of the small fluid segment between the accumulator and the orifice is calculated as

$$L_N = \frac{1}{D_{65}} \quad , \tag{5-10}$$

and its equivalent resistance is

$$R_N = \frac{-D_{66} L_N}{2\dot{W}_N} \quad . \tag{5-11}$$

The accumulator parameters are determined similarly with

$$C_A = \frac{1}{D_{78}} \quad , \tag{5-12}$$

$$L_A = \frac{1}{D_{85}} \quad , \tag{5-13}$$

and

$$R_A = -D_{88} L_A \quad . \tag{5-14}$$

The Nonunique Equilibrium of the Sixth-Order Case

The CTL-V test configuration has some unusual properties if the system is tested without the accumulator. The system no longer possesses a unique equilibrium, but is in equilibrium everywhere that the flows become equal. The system is stable with eigenvalues, as reported earlier, but the system has infinite equilibrium conditions. The sixth order system is described as

$$\dot{x} = \begin{pmatrix} 0 & \frac{1}{C_B} & 0 & -\frac{1}{C_B} & 0 & 0 \\ -\frac{1}{L_L} & -\frac{2R_L}{L_L} DW_{FL} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_B} & D_2 & -D_1 & -D_2 & 0 \\ 0 & 0 & \frac{1}{L_D} & -\frac{2R_D}{L_D} DW_{OS} & -\frac{1}{L_D} & 0 \\ 0 & 0 & 0 & \frac{1}{C_D} & 0 & -\frac{1}{C_D} \\ 0 & 0 & 0 & 0 & \frac{1}{L_N} & -\frac{2R_N}{L_N} DW_{OP2} \end{pmatrix} x . \tag{5-15}$$

Now clearly

$$\Delta\dot{P}_{OS} = \left(\frac{1}{C_B}\right) \Delta F_{FL} - \frac{1}{C_B} \Delta F_{OS} \tag{5-16}$$

is zero if the two flows become equal, and

$$\Delta\dot{F}_{FL} = -\left(\frac{1}{L_L}\right)\Delta P_{OS} - \left(\frac{2R_L}{L_L}\right) DW_{FL}\Delta F_{FL} \tag{5-17}$$

determines the steady-state value of ΔP_{OS} for the nonzero ΔF_{FL}. In the pump equation

$$\Delta\dot{P}_{OD1} = \left(\frac{1}{C_B}\right)\Delta F_{FL} + D_2\Delta P_{OD1} - D_1\Delta F_{OS} - D_2\Delta P_{OI2} \quad , \tag{5-18}$$

if $\Delta\dot{P}_{OD1}$ is zero, then ΔP_{OD1} is related to the other variables as

$$\Delta P_{OD1} = \left(\frac{1}{D_2}\right)\left(D_1\Delta F_{OS} - D_2\Delta P_{OI2} - \left(\frac{1}{C_B}\right)\Delta F_{FL}\right) \quad . \tag{5-19}$$

Proceeding,

$$\Delta\dot{F}_{OS} = \left(\frac{1}{L_D}\right)\Delta P_{OD1} - \left(\frac{2R_D}{L_D} DW_{OS}\right)\Delta F_{OS} - \left(\frac{1}{L_D}\right)\Delta P_{OI2} \quad , \tag{5-20}$$

which, if $\Delta\dot{F}_{OS}$ is zero, similarly can be solved for ΔP_{OD1} as

$$\Delta P_{OD1} = L_D\left(\left(\frac{1}{L_D}\right)\Delta P_{OI2} + \left(\frac{2R_D}{L_D} DW_{OS}\right)\Delta F_{OS}\right) \tag{5-21}$$

The next equation is

$$\Delta\dot{P}_{OI2} = \left(\frac{1}{C_D}\right)(\Delta F_{OS} - \Delta F_{OP2}) \tag{5-22}$$

and finally

$$\Delta\dot{\bar{F}}_{OP2} = \left(\frac{1}{L_N}\right)\Delta P_{OI2} - \left(\frac{2R_N}{L_N} D\bar{W}_{OP2}\right) \Delta F_{OP2} \tag{5-23}$$

which yields a steady-state value for ΔP_{OI2} when all the flows are equal. Notice that by Eqs. (5-19) and (5-21) there are apparently two definitions of ΔP_{OD1}. Both are of the similar form

$$\Delta P_{OD1} = \Delta P_{OI2} + C'\Delta F \tag{5-24}$$

where in Eq. (5-19)

$$C' = \left(\frac{D_1' - \left(\frac{1}{C_B}\right)}{D_2'}\right) \tag{5-25}$$

and in Eq. (5-21)

$$C' = 2R_D\, DW_{OS} \tag{5-26}$$

Interestingly enough for CTL-V at the operating point corresponding to the 100 percent engine power level, one finds that

$$\left(\frac{D_1' - \left(\frac{1}{C_B}\right)}{D_2'}\right) = 2R_D\, DW_{OS} \tag{5-27}$$

Therefore, any time that flows become equal, the system will be in equilibrium at that perturbed condition. This property is due to the particular values of the pump compliance, head rise characteristics, steady-state flow, and the duct resistance.

This result has no meaning in the context of the engine because the engine system has a closed fluid path around these elements, thus altering the overall system dynamic characteristics. Further, if the accumulator is added to the CTL-V system, then the system regains a unique stable equilibrium since the flow into the accumulator must go to zero in the steady state. This response poses no real problem to the technique of this research since the system is driven in an oscillatory fashion about the null, as is true of CTL-V itself.

Results and Conclusions

The most significant result is the ability of the observers or state reconstructors to follow the small signal nonlinear signal even though only an estimate of the system is known. Results in Tables 5-4, 5-5, and 5-6 demonstrate that even with estimates that are in error by large amounts, the robust observer provides reasonable estimates of the state for use in the estimation process. The nonlinear small signal values are the oscillations about the system operating point. The linear values are results from an analytic linearization of the nonlinear system equations. The first group of numbers is the linearized system Jacobian. This matrix is the analytic linearization of the nonlinear equations at the system operating point. The second group of numbers is the result of the estimation process at an instant of time, shown as the first number in the third group. The first two matrices may be compared by positions. The first row of eight numbers in the third group are the nonlinear states described in the first line separated by comas. The remaining rows are as described above. The constrained results of Table 5-6 refer to the method of parameter calculation. Parameter recalculations are permitted only for those elements that, due to model structure, are dependent. That is, accumulator parameters are not permitted to be a function of line or pump. The unconstrained estimation allows variations as with any sensitivity technique. These differences may be observed in the tables as the estimate of the system Jacobian. These observers are not simply following the system but are reconverging after each reevaluation of parameters each 0.02 sec. Since the highest observer root is 200 Hz and the discharge has a resonance at 60 Hz, 2 to 4 sec is a very long run time. Either of these strategies work. The constrained method is similar to a steepest descent technique. The feedback gain must be small, for the constrained approach, to maintain computational stability.

Figures 5-1 through 5-9 demonstrate the observer response. Notice that the pressure initial conditions are presumed known while there is error in the flow initial conditions. The observer response quickly eliminates the flow errors before the estimation process begins. This is a consideration in specifying a settling time. Figure 5-5 is a blowup of Figure 5-4 better displaying the observer response. The four second time histories of Figures 5-1 to 5-9 are to demonstrate that the observer response does not diverge over a long time interval, so that reasonable estimates of the unmeasured states are available for long time periods.

Figures 5-10 and 5-11 show that the linear response is different from the small signal nonlinear response. Recall that the observer attempts to follow the small signal nonlinear response.

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TABLE 5-4. OBSERVER PERFORMANCE AFTER 1.01 sec (UNCONSTRAINED)

LINEARIZED SYSTEM JACOBIAN

.00000000E 00	.14285715E 03	.00000000E 00	-.14285715E 03	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
-.20000000E 01	-.10566397E 01	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.14285715E 03	-.32846130E 02	-.95302567E 02	.32846130E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.76923065E 02	-.11136920E 03	-.76923066E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.12499997E 03	.00000000E 00	-.12499997E 03	.00000000E 00	-.12499997E 03
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.33333325E 03	-.44026642E 02	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.17857132E 02
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.10000000E 04	.00000000E 00	-.10000000E 04	-.20000000E 03

ESTIMATE OF THE SYSTEM JACOBIAN

-.17288065E 01	.14368694E 03	.30166903E 01	-.14781718E 03	.41572742E 01	.12100868E 01	.85676737E 01	-.20503879E-01
-.25805092E 01	-.57489502E 00	.81384474E 00	-.17048140E 01	.20071592E 01	.37530899E 00	.37727804E 01	-.34309067E-02
.94566691E 00	.14416277E 03	-.38617737E 02	-.84466751E 02	.31395279E 02	-.32953844E 01	-.90099487E 01	-.35712200E 00
.20404583E 00	.54084782E-01	.76699371E 02	-.11111844E 03	-.77565689E 02	-.26875138E 00	.54656333E 00	-.12646794E 00
.13345594E 01	.18837857E 00	-.28174248E 01	.12883383E 03	.40882636E 00	-.12679242E 03	-.39272518E 01	-.12604549E 03
-.42008515E-01	.25658238E 00	-.53555202E 00	.11426821E 01	.33235474E 03	-.44417648E 02	.14809996E 00	.36327969E-01
-.14733148E 00	-.80951393E-01	.50218266E 00	-.89543301E 00	.30057430E-02	.39247555E 00	.62844890E 00	.18098373E 02
.42922378E-01	-.32272797E-01	.11512822E 00	-.43026787E 00	.99981836E 03	.15793008E 00	-.99956445E 03	-.19989780E 03

TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PRESS.,DISCH.FLOW,ACCUM. PRESS.,ACCUM. FLOW

1.009982

.13140604E 03	.63241699E 03	.49716895E 03	.63234766E 03	.41520233E 02	.63281616E 03	.41588959E 02	-.45355815E 00

NONLINEAR SMALL SIGNAL STATES

-.15939636E 01	-.68310547E 00	-.14311523E 01	-.75244141E 00	-.15975952E 00	-.28393555E 00	-.91033936E-01	-.45555812E 00

LINEARIZED STATES

-.18141975E 01	-.70624930E 00	-.14839334E 01	-.77351081E 00	-.16730791E 00	-.30493790E 00	-.98584890E-01	-.45318341E 00

OBSERVER STATES

-.15942526E 01	-.67525119E 00	-.14312658E 01	-.74313527E 00	-.16026890E 00	-.26538175E 00	-.91091812E-01	-.45525718E 00

TABLE 5-5. OBSERVER PERFORMANCE AFTER 1.36 sec (UNCONSTRAINED)

LINEARIZED SYSTEM JACOBIAN

.00000000E 00	.14285715E 03	.00000000E 00	-.14285715E 03	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
-.20000000E 01	-.10566397E 01	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.14285715E 03	-.32846130E 02	-.95302567E 02	.32846130E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.76923065E 02	-.11136920E 03	-.76923065E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.12499997E 03	.00000000E 00	-.12499997E 03	.00000000E 00	-.12499997E 03
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.33333325E 03	-.44026642E 02	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.17857132E 02
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.10000000E 04	.00000000E 00	-.10000000E 04	-.20000000E 03

ESTIMATE OF THE SYSTEM JACOBIAN

-.16059237E 01	.14167133E 03	.49668083E 01	-.14946997E 03	.19898062E 01	.15043421E 01	.63707590E 01	.15920824E 00
-.25316229E 01	-.10965233E 01	.13439798E 01	-.21513556E 01	.33961546E 00	.25972545E 00	.47790413E 01	.11894041E 00
.76315600E 00	.14490869E 03	-.37692978E 02	-.87789490E 02	.29808594E 02	-.21506157E 01	-.21309671E 01	.17003471E 00
.20812523E 00	.62574267E-01	.76657623E 02	-.11100710E 03	-.77601593E 02	-.36925721E 00	.72245383E 00	-.16673046E 00
.13209782E 01	.68967319E 00	-.32325726E 01	.12948975E 03	.11368036E 01	-.12726915E 03	-.31465139E 01	-.12665437E 03
.68654157E-02	-.61596376E 00	.11711699E 00	.67755777E 00	.33217578E 03	-.44374451E 02	-.11848326E 01	.19088477E 00
-.15018874E 00	.15521640E 00	.12954420E 00	-.50903267E 00	.36837441E 00	.33396327E 00	.67361516E 00	.18195251E 02
.43917950E-01	.25637436E 00	-.39413053E 00	.16788036E 00	.10004876E 04	-.33912838E-01	-.99945679E 03	-.19984862E 03

TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PRESS.,DISCH.FLOW,ACCUM. PRESS.,ACCUM. FLOW

1.359790

.13505984E 03	.63382690E 03	.50011548E 03	.63387671E 03	.41852875E 02	.63339893E 03	.41782166E 02	.46041393E 00

NONLINEAR SMALL SIGNAL STATES

.20598450E 01	.72680664E 00	.15153809E 01	.77661133E 00	.17288208E 00	.29882813E 00	.10217285E 00	.45841390E 00

LINEARIZED STATES

.18297644E 01	.72499245E 00	.14991608E 01	.77416688E 00	.16740125E 00	.29610878E 00	.96511364E-01	.46078163E 00

OBSERVER STATES

.20595894E 01	.73450249E 00	.15157499E 01	.78475755E 00	.17236006E 00	.31764233E 00	.10214275E 00	.45816791E 00

ORIGINAL PAGE IS
OF POOR QUALITY

TABLE 5-6. OBSERVER PERFORMANCE AFTER 0.81 sec (CONSTRAINED)

LINEARIZED SYSTEM JACOBIAN

.00000000E 00	.14285715E 03	.00000000E 00	-.14285715E 03	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
-.20000000E 01	-.10566397E 01	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.14285715E 03	-.32846130E 02	-.95302567E 02	.32846130E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.76923065E 02	-.11136920E 03	-.76923065E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.12499997E 03	.00000000E 00	-.12499997E 03	.00000000E 00	-.12499997E 03
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.33333325E 03	-.44026642E 02	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.17857132E 02
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.10000000E 04	.00000000E 00	-.10000000E 04	-.20000000E 03

ESTIMATE OF THE SYSTEM JACOBIAN

.00000000E 00	.14316574E 03	.00000000E 00	-.14315262E 03	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
-.26056232E 01	-.97108060E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.14321753E 03	-.33802292E 02	-.93600067E 02	.32647186E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.76856461E 02	-.11115251E 03	-.77164948E 02	.00000000E 00	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.13208276E 03	.00000000E 00	-.12595163E 03	.00000000E 00	-.12553490E 03
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.33287866E 03	-.44101639E 02	.00000000E 00	.00000000E 00
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.17990005E 02
.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.99996997E 03	.00000000E 00	-.98991870E 03	-.19998151E 03

TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PRESS.,DISCH.FLOW,ACCUM. PRESS.,ACCUM. FLOW

.809990

.13141902E 03	.63244116E 03	.49720288E 03	.63236743E 03	.41522827E 02	.63283350E 03	.41590912E 02	-.45155698E 00

NONLINEAR SMALL SIGNAL STATES

-.15809784E 01	-.65893555E 00	-.13972168E 01	-.73266602E 00	-.15716553E 00	-.26660156E 00	-.89080811E-01	-.45355695E 00

LINEARIZED STATES

-.17954273E 01	-.69393921E 00	-.14684181E 01	-.76589817E 00	-.16639012E 00	-.29978377E 00	-.98245084E-01	-.45123541E 00

OBSERVER STATES

-.15807781E 01	-.64633089E 00	-.13968344E 01	-.72222203E 00	-.15866244E 00	-.26919931E 00	-.89261115E-01	-.45961738E 00

Figures 5-12 through 5-17 are a sample of the estimation process response by matrix element. For completeness, Figure 5-14 shows the response of a zero element. All these results are for the eighth order, with accumulator case.

Now examine results of the sixth order, without accumulator, configuration.

Figures 5-18 through 5-23 show the response of the sixth order observer for the varying estimates of the parameters. Again the nonlinear small signal is being followed by a linear observer. Figures 5-24 and 5-25 show the response of the nonlinear small signal states contrasted with the linear response. The estimation response for the sixth order example is demonstrated by Figures 5-26 through 5-29. All of the responses were excited by a 10 Hz perturbation with an amplitude of 27.5 lb/in.^2.

As was anticipated, the technique is fraught with sensitivity, numerical, and dynamic difficulties. The observer must be properly designed with respect to the observed system. Appropriate time intervals must be chosen to allow different dynamics to settle before the parameter estimation process begins. Sample rates must be chosen so that there is numerically sufficient change in variables, providing well conditioned matrices in the calculation process. Gains must be chosen properly to achieve adequate evaluation stability in the parameter estimation process. These considerations can be formidable, especially for systems possessing a large span in eigenvalues. Trial and error generally provide an adequate performance index to determine the times, gains, and sampling intervals. The technique possesses many shortcomings in terms of implementation and system dependence. However, the system has been demonstrated to work for the CTL-V configuration, and the technique has many advantages in application to systems whose states cannot be directly measured.

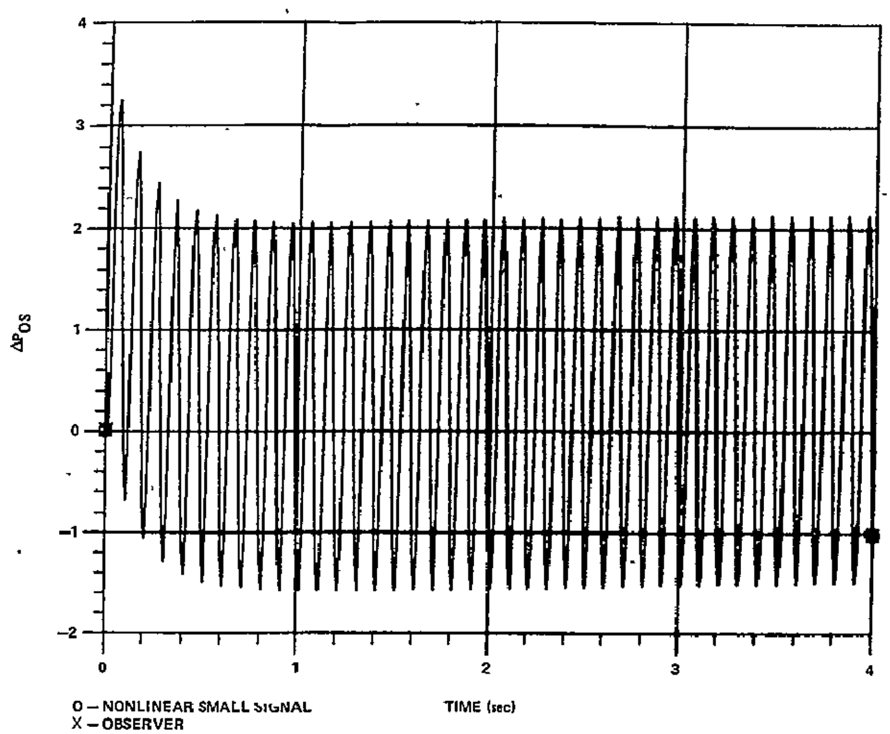

Figure 5-1. Eighth order observer response for ΔP_{OS}.

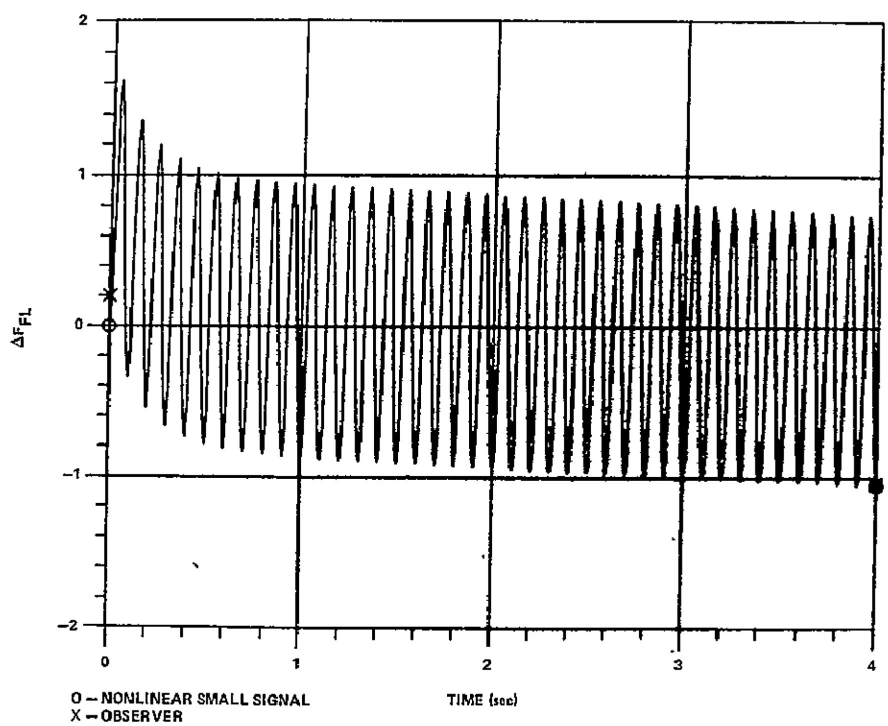

Figure 5-2. Eighth order observer response for ΔF_{FL}.

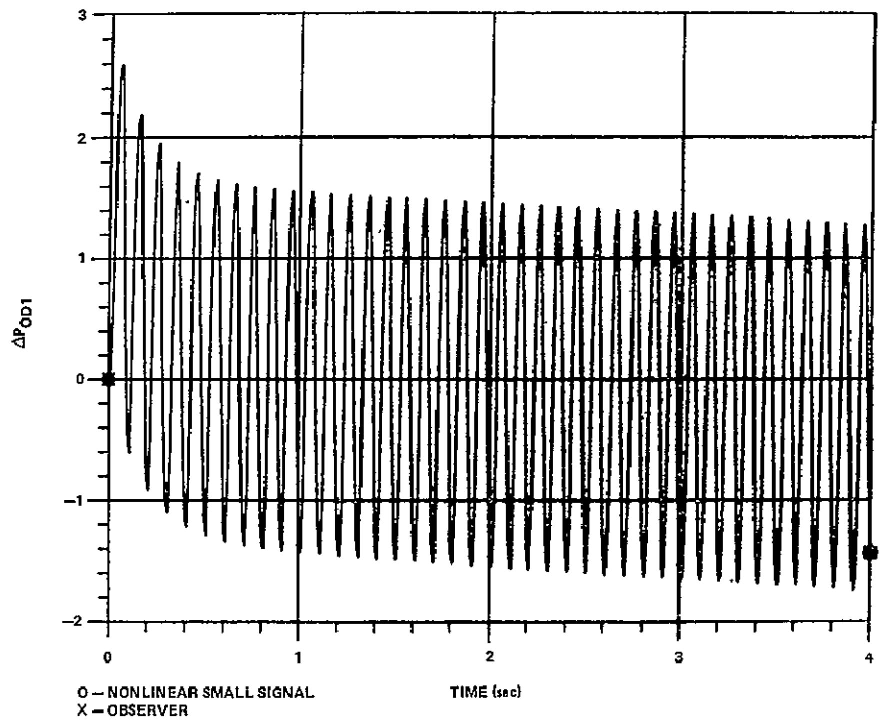

Figure 5-3. Eighth order observer response for ΔP_{OD1}.

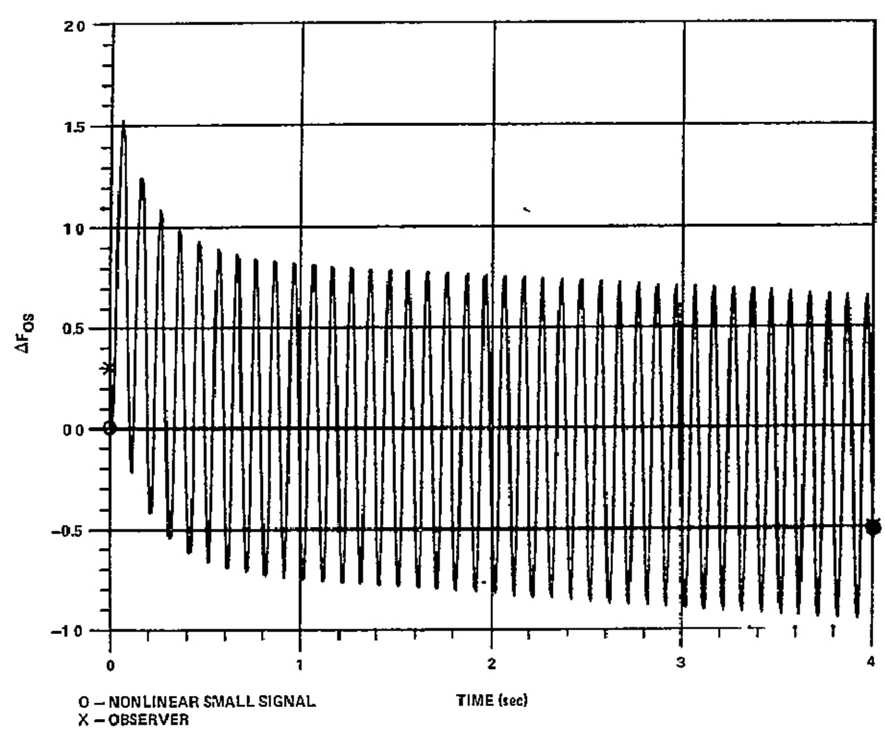

Figure 5-4. Eighth order observer response for ΔF_{OS}.

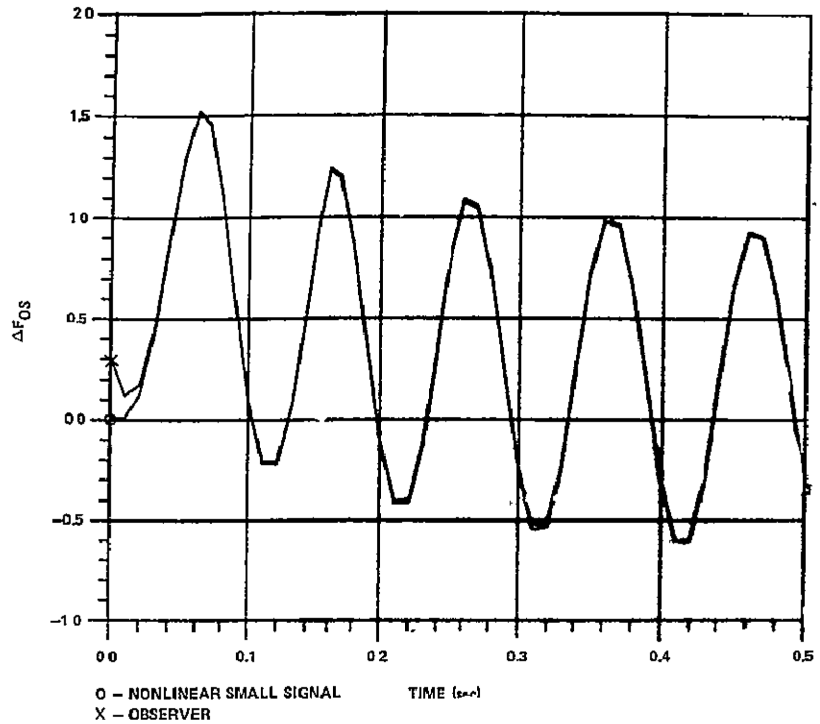

Figure 5-5. Eighth order observer response for ΔF_{OS} (enlarged).

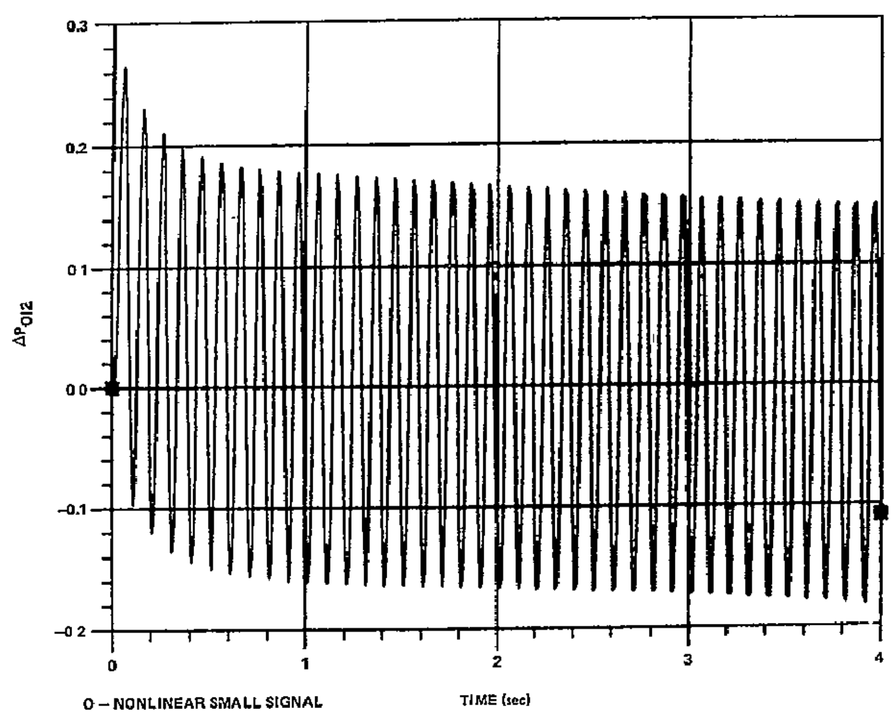

Figure 5-6. Eighth order observer response for ΔP_{OI2}.

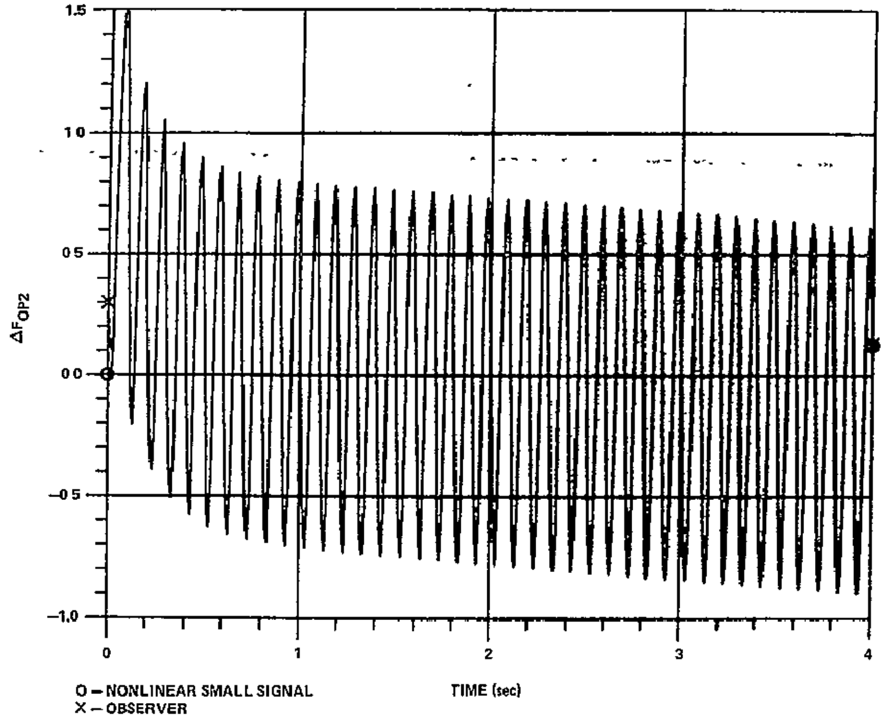

Figure 5-7. Eighth order observer response for ΔF_{OP2}.

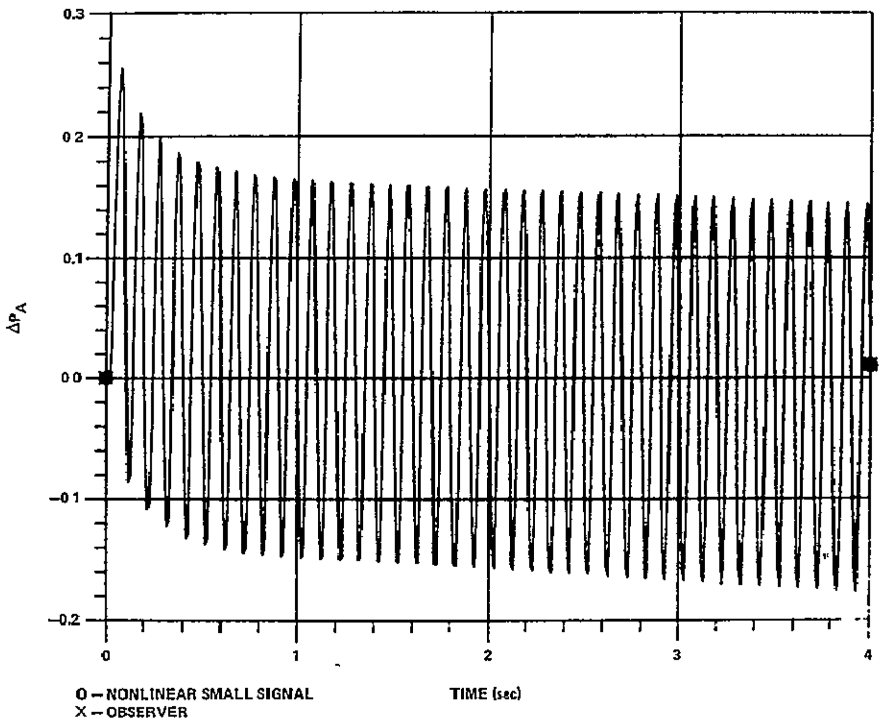

Figure 5-8. Eighth order observer response for ΔP_A.

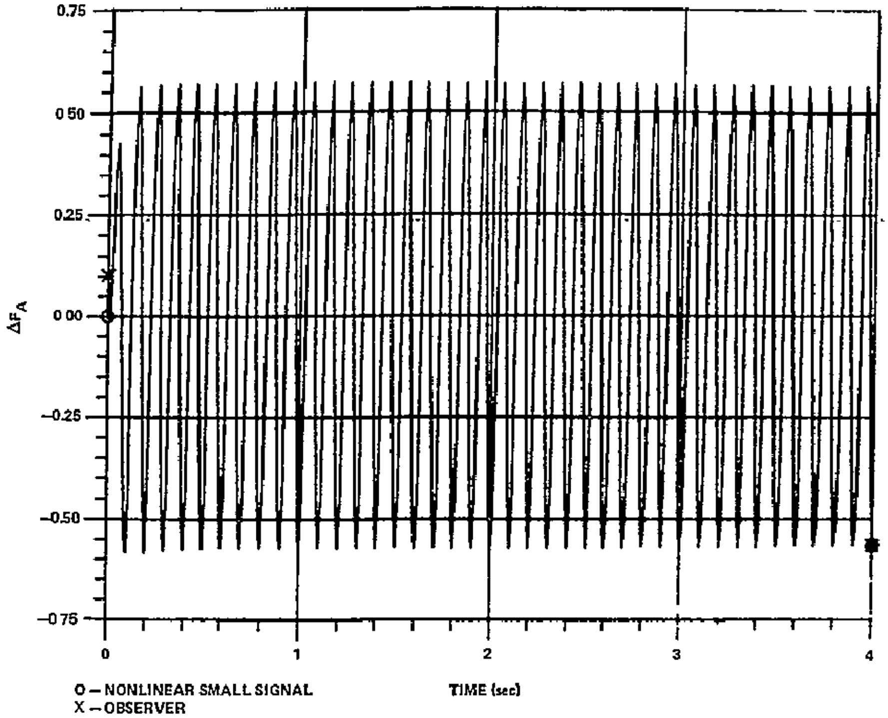

Figure 5-9. Eighth order observer response for ΔF_A.

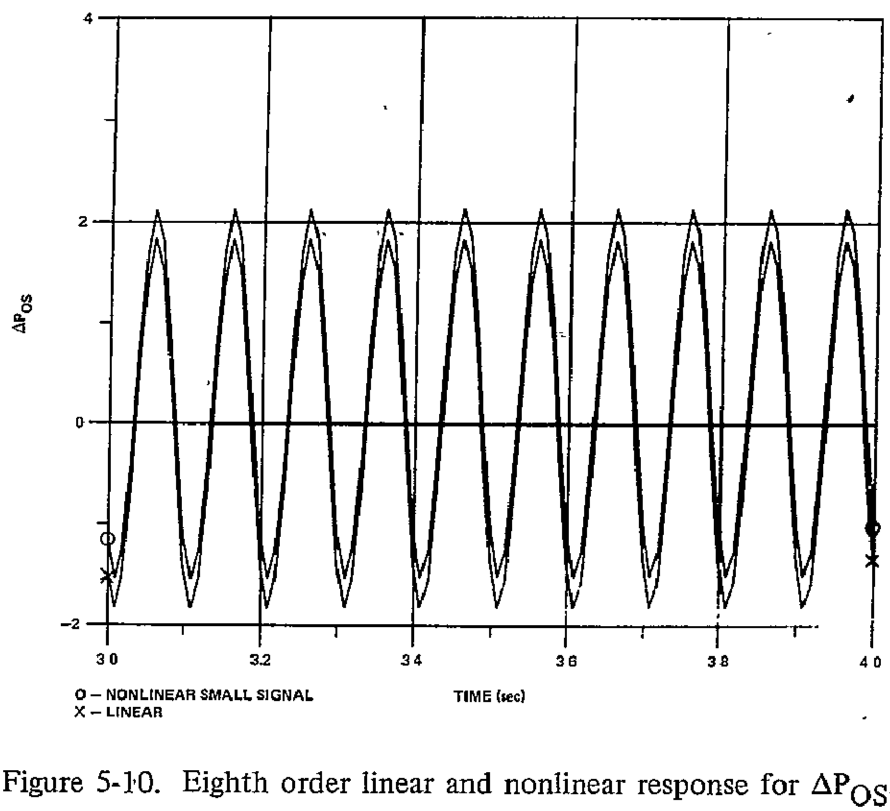

Figure 5-10. Eighth order linear and nonlinear response for ΔP_{OS}.

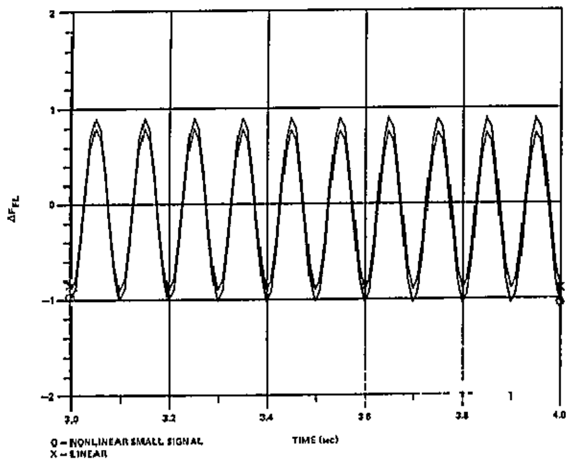

Figure 5-11. Eighth order linear and nonlinear response for ΔF_{FL}.

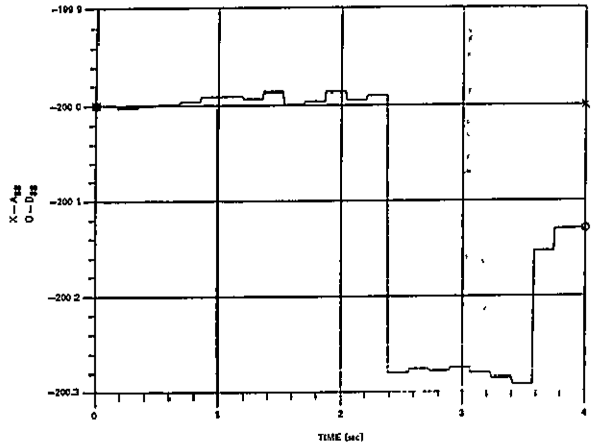

Figure 5-12. Eighth order estimation response for D_{88}.

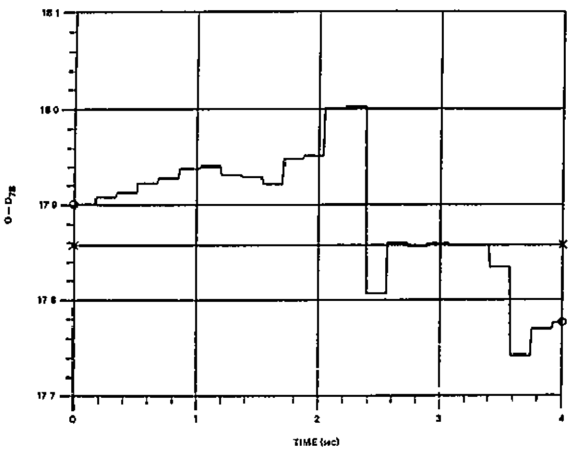

Figure 5-13. Eighth order estimation response for D_{78}.

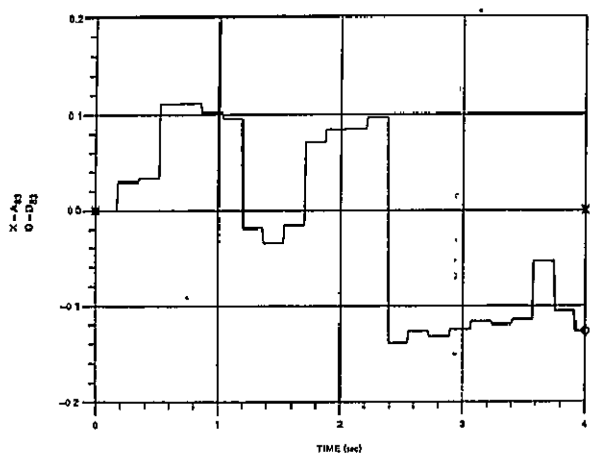

Figure 5-14. Eighth order estimation response for D_{83}.

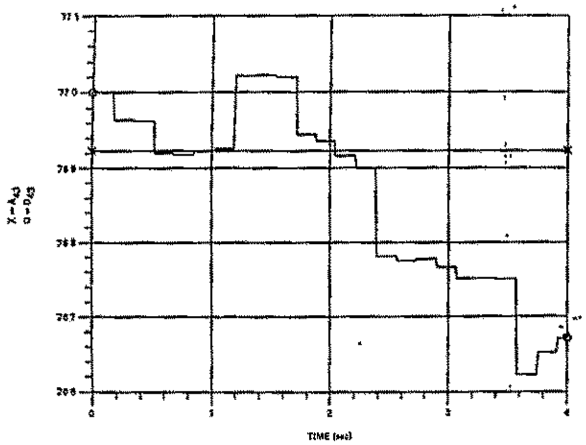

Figure 5-15. Eighth order estimation response for D_{43}.

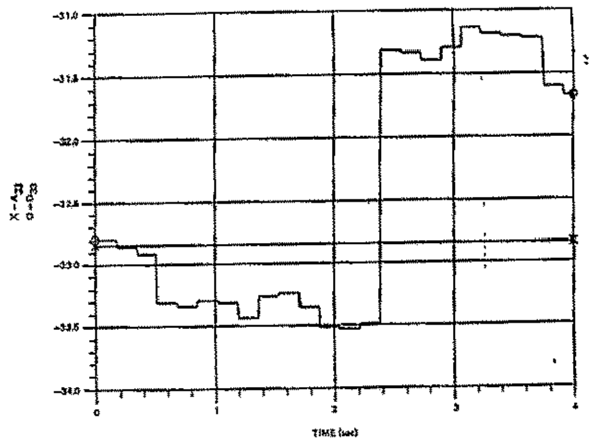

Figure 5-16. Eighth order estimation response for D_{33}.

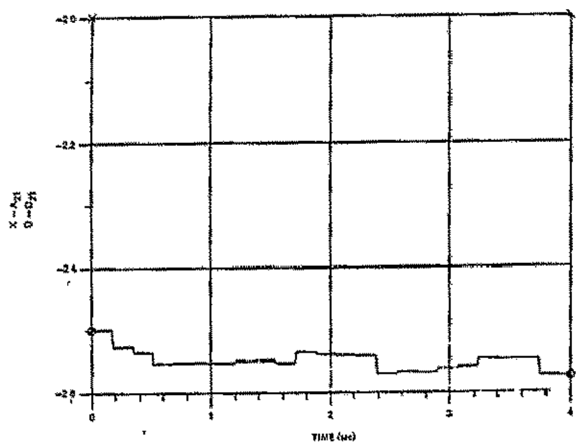

Figure 5-17. Eighth order estimation response for D_{21}.

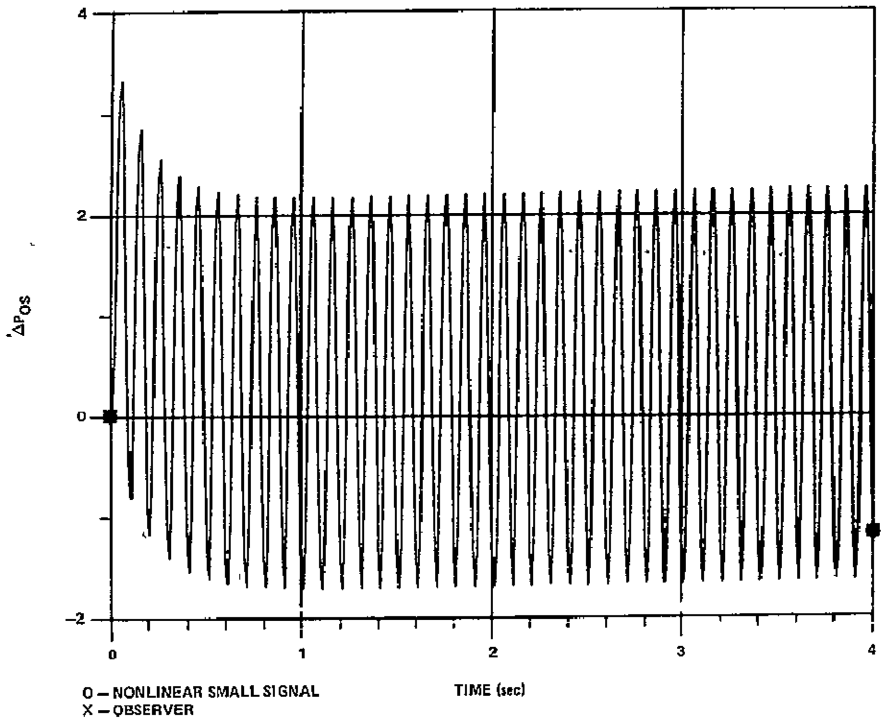

Figure 5-18. Sixth order observer response for ΔP_{OS}.

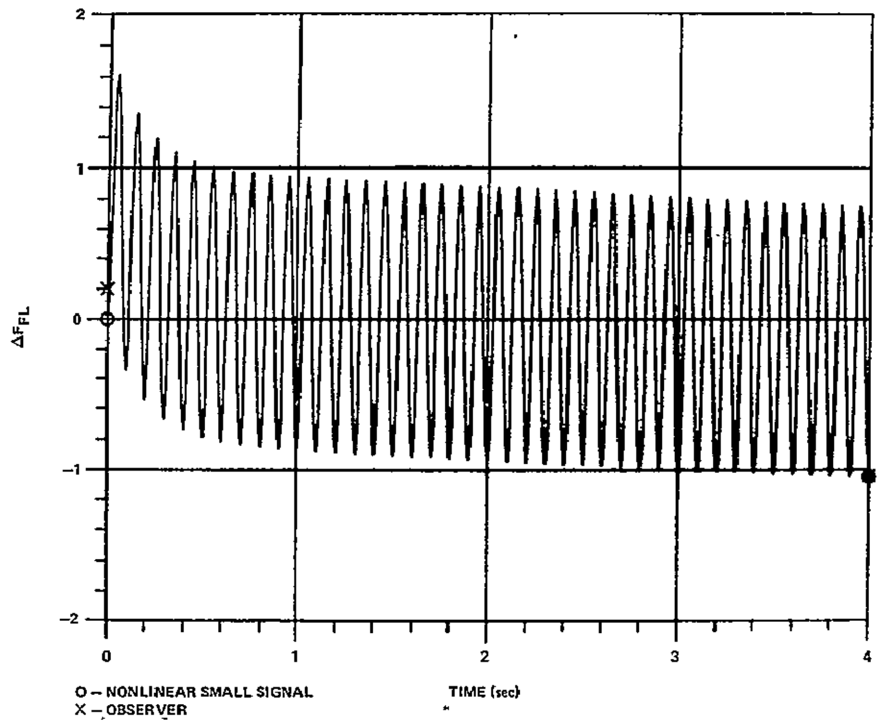

Figure 5-19. Sixth order observer response for ΔF_{FL}.

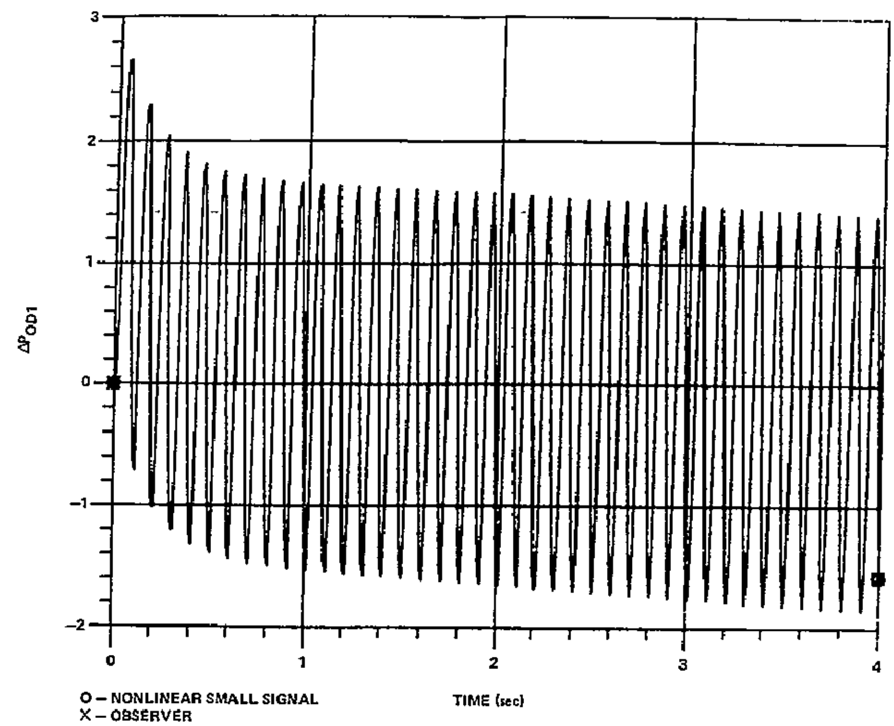

Figure 5-20. Sixth order observer response for ΔP_{OD1}.

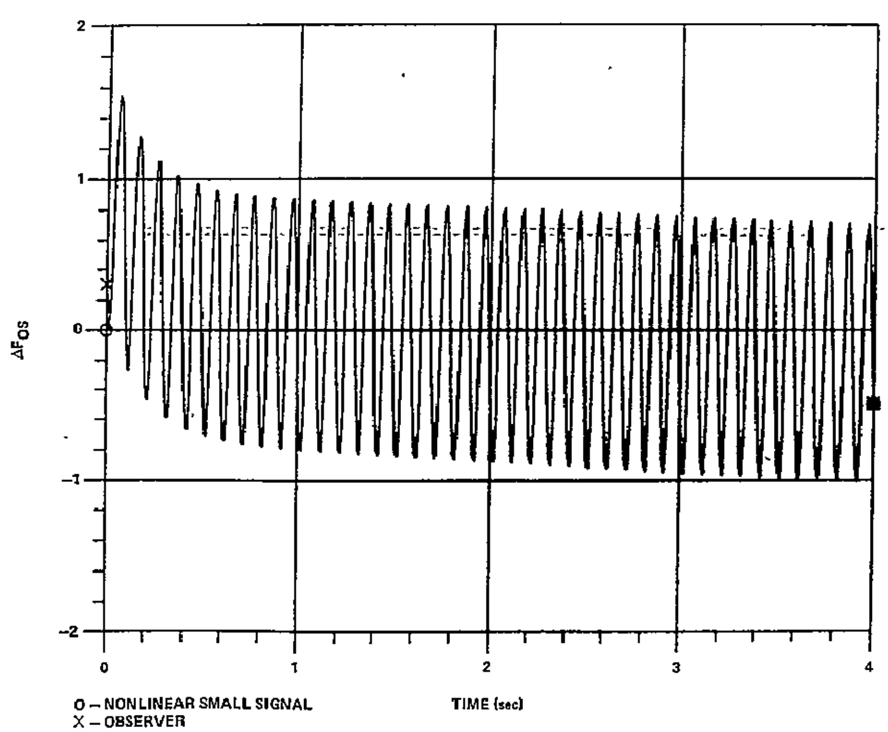

Figure 5-21. Sixth order observer response for ΔF_{OS}.

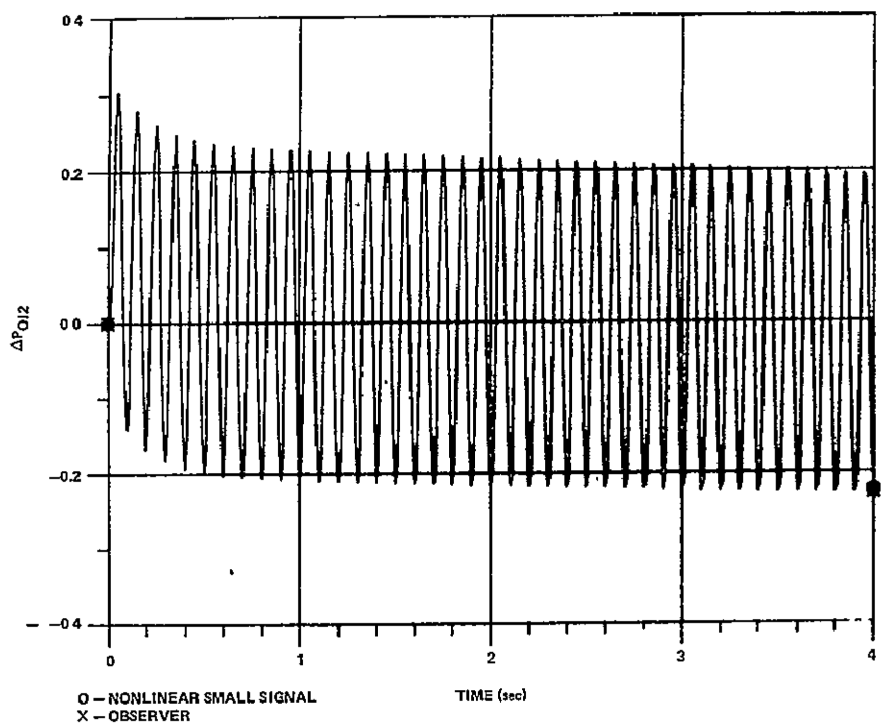

Figure 5-22. Sixth order observer response for ΔP_{OI2}.

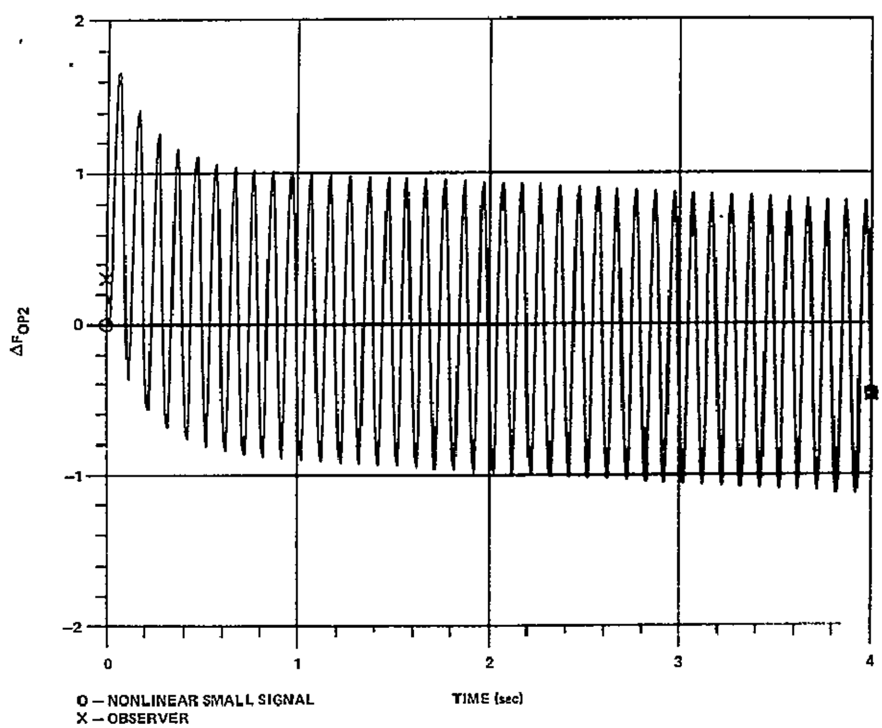

Figure 5-23. Sixth order observer response for ΔF_{OP2}.

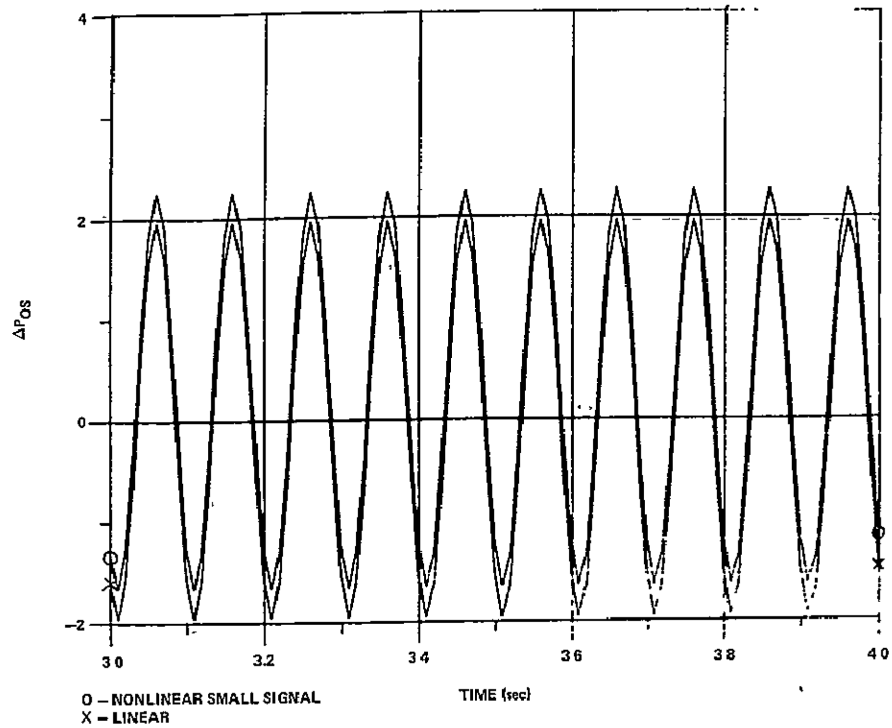

Figure 5-24. Sixth order linear and nonlinear response for ΔP_{OS}.

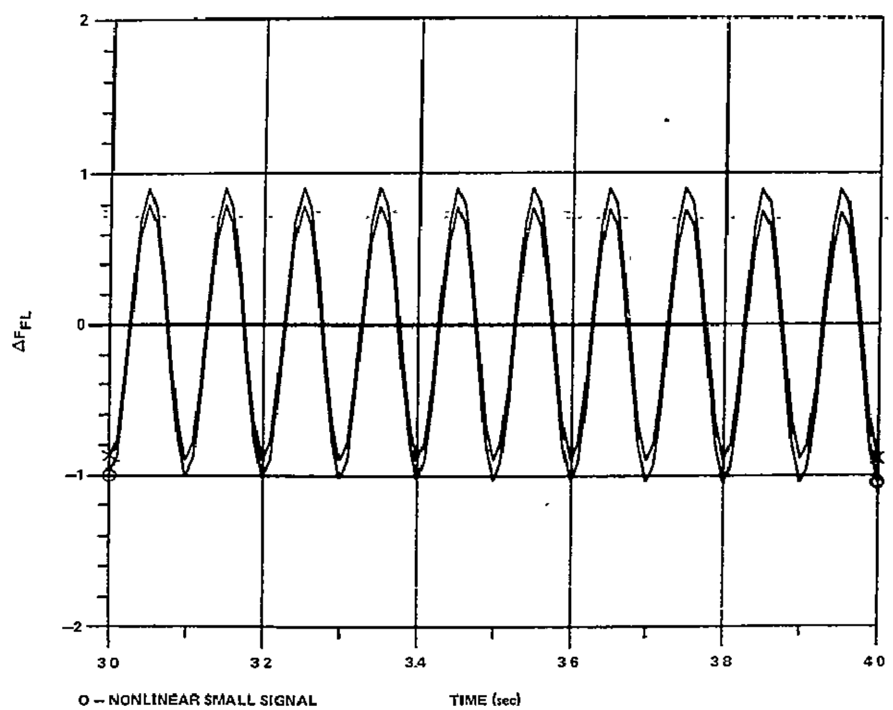

Figure 5-25. Sixth order linear and nonlinear response for ΔF_{FL}.

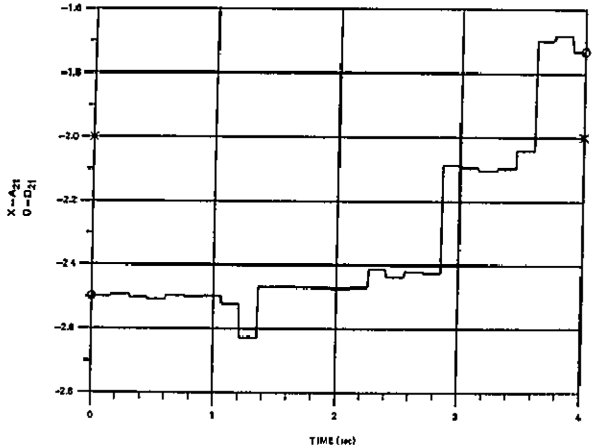

Figure 5-26. Sixth order estimation response for D_{21}.

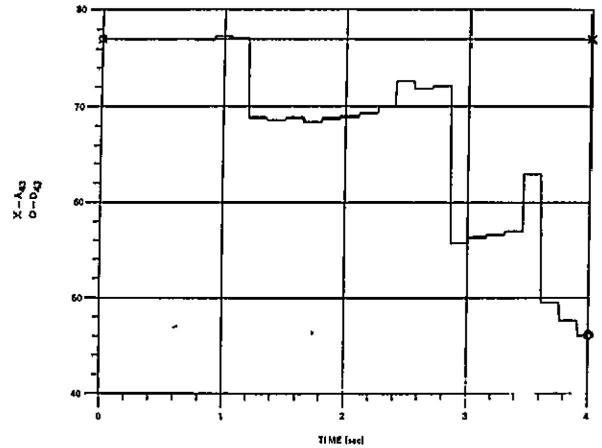

Figure 5-27. Sixth order estimation response for D_{43}.

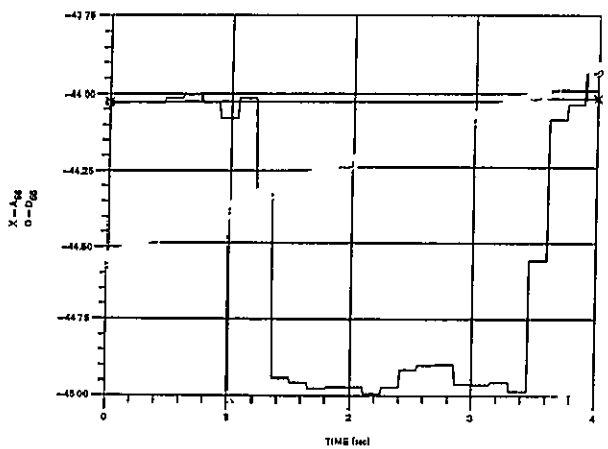

Figure 5-28. Sixth order estimation response for D_{66}.

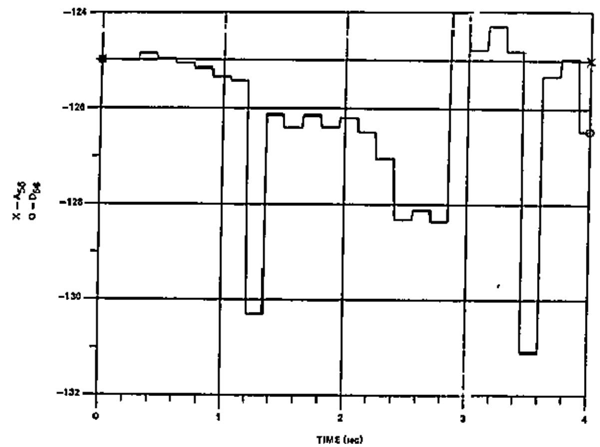

Figure 5-29. Sixth order estimation response for D_{56}.

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APPENDIX A

CTL-V SIMULATION WITH CROSS REFERENCE

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C      CTLV SIMULATION AND PARAMETER ESTIMATION .
C      RUN OPTIONS
C     .KL=0-OBSERVER ONLY,,,ISW=1-ZEROS UNAFFECTED ELEMENTS OF DA,,,
C    ILN SETS SIGNAL FOR OBSERVER TO FOLLOW---0 FOR LIN---1 FOR NONLIN.
C      PRINT OPTIONS  .
C      IPS=0-PRINTS T,X
C  IPS=1-PRINTS T,XLN,XL,XT
C  IPS=2-PRINTS T,XLN,XL,XT,XM,..........IPS1=1-PRINTS ALL TO 6(FILE)
C IPSC=0-PRINTS D,,,,,,IPSC=1-PRINTS D,ME,MX,DA
C    IPSC=2-PRINTS D,ME,MX,DA,MXIN,MX*MXIN
C LK-DISCRETIZATION INTERVAL,,,,KA-SETTLING TIME
         REAL K,ME,MX,MXIN,KC,LL,LC,LN,LA
         DIMENSION A(8,8),C(8,8),ME(8,8),MX(8,8),MXIN(8,8),G(8,8),
     1   K(8,4),C(4,8),DA(8,8),X(8),XD(8),XDL(8),XL(8),XCT(8),XT(8),
     2   XDM(8),XM(8),YT(4),Y(4),ER(8),ERD(8),E(8),ED(8),R(8,8),
     3   D1(8),D2(8),D3(8),D4(8),Z1(4,8),Z2(4,8),Z3(4,8),Z4(4,8),
     4   DD1(8),DD2(8),DD3(8),DD4(8),XLN(8),KC(8,8),DE(8),P(8),XS(8)
C  ZERO OF SIGMA USED MEMORY
C   REMOVE FOR USE ON SYSTEMS OTHER THAN XEROX
         GO TO 800
  10     CONTINUE
         READ(1,3)IPS,IPSO,IPS1,ISW,ILN
 3       FORMAT(5I3)
         READ(1,9)NN,MM,LK,KL,KA,CT,AA,W,AGN
 2       FORMAT(1H1,'CTL-V'/'OUTPUT OPTIONS'/5I5/
     1   'SYSTEM ORDER,MEASUREMENTS,DISCR. INT.,OBS. SW.,SETLNG TM'
     1   /5I5/'DELT,INPUT AMP.,INPUT FREQ.,FEEDBACK GAIN'
     2   /4E20.8/'PUMP COMP.,LINE INERT.,LINE RESIST.,PUMP SPEED'/4E20.8/
     3   'PUMP CONST.,PUMP SLOPE,DUCT COMP.,DUCT INERT.'/4E20.8/
     4   'DUCT RESIST.,NOM. LINE FLOW,NOM. DUCT FLOW,NOM. DISCH. FLOW'/
     5   4E20.8/'DISCH. INERT.,DISCH. RESIST.,TIME,ACCUM. COMP.'/4E20.8/
     6   'ACCUM. INERT.,ACCUM. RESIST.,'/2E20.8//
     6   'LIMIT TIME,INPUT PRESS.,DISCH. PRESS.'/3E20.8//
     1   'BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW'
     7   /'DISCH. PRESS.,DISCH. FLOW,ACCUM. PRESS.,ACCUM. FLOW'
     8       /4E20.8/4E20.8/'LINEARIZED STATES'/4E20.8/4E20.8/
     9   'OBSERVER STATES'/4E20.8/4E20.8/'MODEL STATES'/4E20.8/4E20.8/
     6   'OBSERVER GAIN MATRIX'/4E20.8/4E20.8/4E20.8/4E20.8/4E20.8/4E20.8/4
     1   E20.8/4E20.8//'MEASUREMENT MATRIX'/8E15.8/8E15.8/8E15.8/8E15.8
     2   //'NOM. TEST COND.'/8E15.8//
     3   'ESTIMATE OF SYSTEM JACOBIAN'/(8E15.8))
         READ(1,1)CB,LL,RL,SO1,CON,PGPH,LD,RD,X4N,X2N,CD,LN,RN,X6N
     1   ,CA,LA,RA,TLM,PPT,POP2,T,DU1,DU2,DU3
     2   ,(X(I),I=1,NN,1),(XL(I),I=1,NN,1),(XT(I),I=1,NN,1),(XM(I),I=1,NN,1
     3   ),((K(I,J),I=1,NN,1),J=1,MM,1),((C(I,J),I=1,MM,1),J=1,NN,1),
     4   (XS(I),I=1,8),((D(I,J),I=1,8),J=1,8)
 1       FORMAT (4E20.8)
         WRITE(5,2)IPS,IPSC,IPS1,ISW,ILN,
     1           NN,MM,LK,KL,KA,CT,AA,W,AGN,CB,LL,RL,SO1,CON,PGPH,CD,LD,R
     1   D,X2N,X4N,X6N,LN,RN,T,CA,LA,RA,TLM,PPT,POP2,(X(I),I=1,8),(XL(I),I=
```

```
     2 1,8),(XT(I),I=1,8),(XM(I),I=1,8),((K(I,J),J=1,4),I=1,8),
     3 ((C(I,J),J=1,8),I=1,4),(XS(I),I=1,8)
     4 ,((D(I,J),J=1,8),I=1,8)
    9  FORMAT(5I3/(4E20.8))
C
       NL=0
       DO 30 I=1,NN
       XIN(I)=X(I)-XS(I)
   30  CONTINUE
       KB=KA
C
       DO 50 I=1,8
       DO 40 J=1,8
   40  A(I,J)=0.0
   50  CONTINUE
       A(1,2)=1./CB
       A(1,4)=-A(1,2)
       A(2,1)=-1./LL
       A(2,2)=-2.*RL*X2N/LL
       A(3,2)=1./CB
       A(3,3)=(CON*SO1*PGPH)/LD
       A(3,4)=-2.*A(3,3)*RD*X4N-A(3,2)
       A(3,5)=-A(3,3)
       A(4,3)=1./LD
       A(4,4)=-2.*RD*X4N/LD
       A(4,5)=-A(4,3)
       A(5,4)=1./CD
       A(5,6)=-A(5,4)
       A(5,8)=A(5,6)
       A(6,5)=1./LN
       A(6,6)=-2.*RN*X6N/LN
       A(7,8)=1./CA
       A(8,5)=1./LA
       A(8,7)=-A(8,5)
       A(8,8)=-RA/LA
C
       WRITE(5,5)((A(I,J),J=1,8),I=1,8)
    5  FORMAT(//'LINEARIZED SYSTEM JACOBIAN'/(8E15.8))
C
C
C
       JJ=0
C    KC MATRIX CALCULATION
       DO 120 I=1,8
       DO 110 J=1,8
       KC(I,J)=0.0
       DO 99 L=1,4
   99  KC(I,J)=KC(I,J)+K(I,L)*C(L,J)
  110  CONTINUE
  120  CONTINUE
C
```

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```
          IF (IFS1.NE.1)GØ TØ 190
          WRITE(6)T,(X(I),I=1,NN),(XLN(I),I=1,NN),(XL(I),I=1,NN),(XT(I),I=1,
        1 NN),(XM(I),I=1,NN,1),((D(I,J),I=1,NN,1),J=1,NN,1)
        2 ,((A(I,J),I=1,NN,1),J=1,NN,1)
  190     WRITE(5,7)T,(X(I),I=1,8),(XLN(I),I=1,8),
        1 (XL(I),I=1,8),(XT(I),I=1,8)
  200     CØNTINUE
C
C
C  ACCELERATIØN CALCULATIØN
          T=T+DT
          JK=0
          D5=T-DT
  300     DØ 310 I=1,NN
          D1(I)=X(I)
          D2(I)=XL(I)
          D3(I)=XT(I)
          D4(I)=XM(I)
  310     CØNTINUE
          PT=AA*SIN(W*D5)
          DD1(1)=(D1(2)-D1(4))/CB
          DD1(2)=((PPT-D1(1))-RL*D1(2)*D1(2))/LL+PT/LL
          DD1(3)=DD1(1)+(CØN*SA1*FGPH/LD)*(D1(3)-D1(5)-RD*D1(4)**2)
          DD1(4)=(D1(3)-D1(5)-RD*D1(4)*D1(4))/LD
          DD1(5)=(D1(4)-D1(6)-D1(8))/CD
          DD1(6)=(D1(5)-PCP2-RN*D1(6)*D1(6))/LN
          DD1(7)=(D1(8)/CA)
          DD1(8)=(D1(5)-D1(7)-RA*D1(8))/LA
          DØ 330 I=1,NN
          DD2(I)=0.
          DD3(I)=0.
          DD4(I)=0.
          DØ 320 J=1,NN
          DD2(I)=DD2(I)+A(I,J)*D2(J)
          IF (ILN.NE.0)GØ TØ 315
          DD3(I)=DD3(I)+D(I,J)*D3(J)+KC(I,J)*(D2(J)-D3(J))
  315     IF (ILN.EQ.1)DD3(I)=DD3(I)+D(I,J)*D3(J)+KC(I,J)*(D1(J)-XS(J)-D3(J)
        1 )
  320     DD4(I)=DD4(I)+D(I,J)*D4(J)
  330     CØNTINUE
          DD2(2)=DD2(2)+PT/LL
          DD3(2)=DD3(2)+PT/LL
          DD4(2)=DD4(2)+PT/LL
          IF (JK.NE.0) GØ TØ 380
          JK=JK+1
  340     DØ 370 I=1,NN
          Z1(JK,I)=DT*DD1(I)
          Z2(JK,I)=DT*DD2(I)
          Z3(JK,I)=DT*DD3(I)
          Z4(JK,I)=DT*DD4(I)
          IF (JK.GE.4) GØ TØ 360
```

```
      IF (JK.NE.3) GO TO 350
      D1(I)=X(I)+Z1(JK,I)
      D2(I)=XL(I)+Z2(JK,I)
      D3(I)=XT(I)+Z3(JK,I)
      D4(I)=XM(I)+Z4(JK,I)
      D5=T
      GO TO 360
  350 D1(I)=X(I)+Z1(JK,I)/2.
      D2(I)=XL(I)+Z2(JK,I)/2.
      D3(I)=XT(I)+Z3(JK,I)/2.
      D4(I)=XM(I)+Z4(JK,I)/2.
      D5=T-DT/2.
  360 CONTINUE
  370 CONTINUE
      GO TO 310
  380 IF (JK.NE.1) GO TO 390
      JK=JK+1
      GO TO 340
  390 IF (JK.NE.2) GO TO 391
      JK=JK+1
      GO TO 340
  391 IF (JK.NE.3) GO TO 392
      JK=JK+1
      GO TO 340
  392 IF (JK.NE.4) GO TO 394
      JK=JK+1
      DO 393 I=1,NN
      X(I)=X(I)+(Z1(1,I)+2.*Z1(2,I)+2.*Z1(3,I)+Z1(4,I))/6.
      XL(I)=XL(I)+(Z2(1,I)+2.*Z2(2,I)+2.*Z2(3,I)+Z2(4,I))/6.
      XT(I)=XT(I)+(Z3(1,I)+2.*Z3(2,I)+2.*Z3(3,I)+Z3(4,I))/6.
      XM(I)=XM(I)+(Z4(1,I)+2.*Z4(2,I)+2.*Z4(3,I)+Z4(4,I))/6.
  393 CONTINUE
      D5=T
      GO TO 300
  394 DO 395 I=1,NN
      XD(I)=DD1(I)
      XDL(I)=DD2(I)
      XDT(I)=DD3(I)
      XDM(I)=DD4(I)
  395 CONTINUE
C    MEASUREMENTS
      DO 410 I=1,MM
      Y(I)=0.
      YT(I)=0.
      DO 400 J=1,NN
      Y(I)=Y(I)+C(I,J)*XL(J)
      YT(I)=YT(I)+C(I,J)*XT(J)
  400 CONTINUE
  410 CONTINUE
      DO 415 I=1,6
      XLN(I)=X(I)-XS(I)
```

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```
  415      CONTINUE
           NL=NL+1
           IF (NL.EQ.LK)NL=0
           IF (IFS1.NF.1)GO TO 420
           IF (NL.EQ.0)WRITE(6)T,(X(I),I=1,NN),(XLN(I),I=1,NN),(XL(I),I=1,NN)
          1 ,(XT(I),I=1,NN,1),(XM(I),I=1,NN,1),((D(I,J),I=1,NN,1),J=1,NN,1)
          2 ,((A(I,J),I=1,NN,1),J=1,NN,1)
  420      CONTINUE
           IF (IFS.EQ.0)GO TO 421
           IF (IFS.EQ.1)GO TO 422
           IF (IFS.EQ.2)GO TO 423
           GO TO 424
  421      IF (NL.EQ.0)WRITE(5,6)T,(X(I),I=1,8)
           GO TO 424
  422      IF (NL.EQ.0)WRITE(5,7)T,(X(I),I=1,8),(XLN(I),I=1,8),
          1 (XL(I),I=1,8),(XT(I),I=1,8)
           GO TO 424
  423      IF (NL.EQ.0)WRITE(5,8)T,(X(I),I=1,8),(XLN(I),I=1,8)
          1 ,(XL(I),I=1,8),(XT(I),I=1,8),(XM(I),I=1,8)
           GO TO 424
  6        FORMAT(/'TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PR
          1 ESS.,DISC. FLOW,ACCUM. PRESS.,ACCUM. FLOW'/1F10.6/8E15.8)
  7        FORMAT(/'TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PR
          1 ESS.,DISCH. FLOW,ACCUM. PRESS,ACCUM. FLOW'/1F10.6/8E15.8/
          2 'NONLINEAR SMALL SIGNAL STATES'/8E15.8/'LINEARIZED STATES'/8E15.8/
          3 'OBSERVER STATES'/8E15.8/)
  8        FORMAT(/'TIME/BUBBLE PRESS.,LINE FLOW,HEADRISE,DUCT FLOW,DISCH. PR
          1 ESS.,DISCH.FLOW,ACCUM. PRESS.,ACCUM. FLOW'/1F10.6/8E15.8/
          2 'NONLINEAR SMALL SIGNAL STATES'/8E15.8/'LINEARIZED STATES'/8E15.8/
          3 'OBSERVER STATES'/8E15.8/'MODEL STATES'/8E15.8/)
  424      CONTINUE
           IF (KL.EQ.0)GO TO 790
           IF (NL.NE.0)GO TO 790
           KB=KB-1
           IF (KB.EQ.1)GO TO 480
           GO TO 490
  480      DO 485 I=1,NN
           XM(I)=XT(I)
           XDM(I)=XDT(I)
  485      CONTINUE
  490      IF (KB.GT.0)GO TO 790
  C
           IF (JJ.NE.0)GO TO 500
           JJ=NN
  500      J=JJ
           DO 520 I=1,NN
           E(I)=XT(I)-XM(I)
           ER(I)=XL(I)-XM(I)
           ED(I)=XDT(I)-XDM(I)
           ERD(I)=XDL(I)-XDM(I)
           DE(I)=0.0
```

```
      DO 510 KK=1,NN
      DE(I)=DE(I)+D(I,KK)*F(KK)
510   CONTINUE
C
      ME(I,J)=ED(I)-DE(I)
      MX(I,J)=XT(I)
520   CONTINUE
      J=J-1
      JJ=J
      DO 530 I=1,NN
      XM(I)=XT(I)
      XDM(I)=XDT(I)
530   CONTINUE
      IF (J.NE.0)GO TO 790
      J=NN
600   DO 620 I=1,NN
      IF (I.EQ.J)GO TO 610
      P(I)=0.0
      GO TO 620
610   P(I)=1.0
620   CONTINUE
      DO 640 I=1,NN
      DO 630 KK=1,NN
630   B(I,KK)=MX(I,KK)
640   CONTINUE
      DO 635 I=1,NN
635   MXIN(I,J)=0.0
      M=NN-1
      DO 650 I=1,M
      L=I+1
      DO 650 JJ=L,NN
      IF (B(JJ,I).EQ.0.)GO TO 650
      DO 658 KK=L,NN
658   B(JJ,KK)=B(JJ,KK)-B(I,KK)*B(JJ,I)/B(I,I)
      P(JJ)=P(JJ)-P(I)*B(JJ,I)/B(I,I)
650   CONTINUE
      MXIN(NN,J)=P(NN)/B(NN,NN)
      DO 670 I=1,M
      KK=NN-I
      L=KK+1
      DO 660 JJ=L,NN
660   P(KK)=P(KK)-MXIN(JJ,J)*B(KK,JJ)
      MXIN(KK,J)=P(KK)/B(KK,KK)
670   CONTINUE
      J=J-1
      IF (J.NE.0)GO TO 600
      KP=KA
      DO 720 I=1,NN
      DO 710 J=1,NN
      DA(I,J)=0.0
      DO 700 KK=1,NN
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```
      DA(I,J)=DA(I,J)+ME(I,KK)*MXIN(KK,J)
  700 CONTINUE
  710 CONTINUE
  720 CONTINUE
C
      DO 757 I=1,NN
      DO 756 J=1,NN
      G(I,J)=0.
      DO 755 KK=1,NN
  755 G(I,J)=G(I,J)+MX(I,KK)*MXIN(KK,J)
  756 CONTINUE
  757 CONTINUE
      GO TO 759
  758 CONTINUE
      IF (IFSO.EQ.0)WRITE(6,11)((D(I,J),J=1,8),I=1,8)
C
      IF (IFSO.EQ.1)WRITE(6,12)((D(I,J),J=1,8),I=1,8),((ME(I,J),J=1,8),I
     1 =1,8),((MX(I,J),J=1,8),I=1,8),((DA(I,J),J=1,8),I=1,8)
      IF (IFSO.EQ.2)WRITE(6,13)((D(I,J),J=1,8),I=1,8),((ME(I,J),J=1,8),I
     1 =1,8),((MX(I,J),J=1,8),I=1,8),((DA(I,J),J=1,8),I=1,8),
     2 ((MXIN(I,J),J=1,8),I=1,8),((G(I,J),J=1,8),I=1,8)
   11 FORMAT(/'ESTIMATE OF THE SYSTEM JACOBIAN'/(8E15.8))
   12 FORMAT(/'ESTIMATE OF THE SYSTEM JACOBIAN'/8E15.8/8E15.8/8E15.8/
     1 8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//'ERROR MATRIX'/8E15.8/8E15.8
     2 /8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//'STATE MATRIX'/
     3 8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//
     4 'DELTA JACOBIAN'/(8E15.8))
   13 FORMAT(/'ESTIMATE OF THE SYSTEM JACOBIAN'/8E15.8/8E15.8/8E15.8/
     1 8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//'ERROR MATRIX'/8E15.8/8E15.8
     2 /8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//'STATE MATRIX'/
     3 8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//
     4 'DELTA JACOBIAN'/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/8E15.8/
     5 8E15.8/8E15.8//'STATE MATRIX INVERSE'/8E15.8/8E15.8/8E15.8/
     6 8E15.8/8E15.8/8E15.8/8E15.8/8E15.8//'PRODUCT OF MATRIX AND INV.'
     7 /(8E15.8))
      GO TO 790
  759 JJ=0
      IF (ISW.NE.1)GO TO 7588
      DA(1,1)=0.
      DO 7581 I=3,NN
 7581 DA(I,1)=0.
      DO 7582 I=4,NN
 7582 DA(I,2)=0.
      DA(1,3)=0.
      DA(2,3)=0.
      DO 7583 I=5,NN
 7583 DA(I,3)=0.
      DA(2,4)=0.
      DO 7584 I=6,NN
 7584 DA(I,4)=0.
      DA(1,5)=0.
```

```
      DA(2,5)=0.
      DA(5,5)=0.
      DA(7,5)=0.
      DO 7585 I=1,4
 7585 DA(I,6)=0.
      DA(7,6)=0.
      DA(8,6)=0.
      DO 7586 I=1,7
 7586 DA(I,7)=0.
      DO 7587 I=1,4
 7587 DA(I,8)=0.
      DA(6,8)=0.
 7588 DO 770 I=1,NN
      DO 760 J=1,NN
 760  D(I,J)=D(I,J)+DA(I,J)*AGN
 770  CONTINUE
      GO TO 758
 790  CONTINUE
      IF (T.GT.TLM)CALL EXIT
      GO TO 200
    4 FORMAT(/1F10.3/(4E16.8))
C ZERO MEMORY CALL
  800 CALL ZAP(K,M)
      GO TO 10
      END
```

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A	13.000	64.000	66.000	67.000	68.000	69.000	70.000	71.000	72.000
	73.000	74.000	75.000	76.000	77.000	78.000	79.000	80.000	81.000
	82.000	83.000	84.000	85.000	87.000	103.000	135.000	208.000	
AA	24.000	48.000	121.000						
AGN	24.000	48.000	371.000						
B	13.000	278.000	286.000	288.000	289.000	291.000	296.000	297.000	
C	13.000	42.000	48.000	58.000	198.000	199.000			
CA	42.000	48.000	82.000	128.000					
CB	42.000	48.000	66.000	70.000	122.000				
CD	42.000	48.000	77.000	126.000					
CON	42.000	48.000	71.000	124.000					
D	13.000	42.000	48.000	103.000	137.000	138.000	140.000	208.000	256.000
	320.000	322.000	324.000	371.000					
DA	13.000	304.000	306.000	322.000	324.000	344.000	346.000	348.000	349.000
	350.000	352.000	353.000	355.000	356.000	357.000	358.000	359.000	361.000
	362.000	363.000	365.000	367.000	368.000	371.000			
DD1	13.000	122.000	123.000	124.000	125.000	126.000	127.000	128.000	129.000
	148.000	188.000							
DD2	13.000	131.000	135.000	142.000	149.000	189.000			
DD3	13.000	132.000	137.000	138.000	143.000	150.000	190.000		
DD4	13.000	133.000	140.000	144.000	151.000	191.000			
DE	13.000	254.000	256.000	259.000					
DT	24.000	48.000	112.000	114.000	148.000	149.000	150.000	151.000	164.000
DU1	42.000								
DU2	42.000								
DU3	42.000								
D1	13.000	116.000	122.000	123.000	124.000	125.000	126.000	127.000	128.000
	129.000	138.000	154.000	160.000					
D2	13.000	117.000	135.000	137.000	155.000	161.000			
D3	13.000	118.000	137.000	138.000	156.000	162.000			
D4	13.000	119.000	140.000	157.000	163.000				
D5	114.000	121.000	158.000	164.000	185.000				
F	13.000	250.000	256.000						
FD	13.000	252.000	259.000						
FR	13.000	251.000							
FRD	13.000	253.000							
FXIT	375.000								
G	13.000	313.000	315.000	324.000					
I	42.000	48.000	57.000	58.000	62.000	64.000	87.000	94.000	96.000
	98.000	103.000	106.000	115.000	116.000	117.000	118.000	119.000	130.000
	131.000	132.000	133.000	135.000	137.000	138.000	140.000	147.000	148.000
	149.000	150.000	151.000	154.000	155.000	156.000	157.000	160.000	161.000
	162.000	163.000	179.000	180.000	181.000	182.000	183.000	187.000	188.000
	189.000	190.000	191.000	194.000	195.000	196.000	198.000	199.000	202.000
	203.000	208.000	216.000	218.000	221.000	240.000	241.000	242.000	249.000
	250.000	251.000	252.000	253.000	254.000	256.000	259.000	260.000	264.000
	265.000	266.000	270.000	271.000	272.000	274.000	276.000	278.000	280.000
	281.000	283.000	284.000	286.000	288.000	289.000	292.000	293.000	302.000
	304.000	306.000	311.000	313.000	315.000	320.000	322.000	324.000	345.000
	346.000	347.000	348.000	351.000	352.000	354.000	355.000	360.000	361.000
	364.000	365.000	366.000	367.000	369.000	371.000			

ILN	22.000	48.000	136.000	138.000					
IPS	22.000	48.000	212.000	213.000	214.000				
IPSC	22.000	48.000	320.000	322.000	324.000				
IPS1	22.000	48.000	102.000	207.000					
ISW	22.000	48.000	343.000						
J	42.000	48.000	63.000	64.000	87.000	95.000	96.000	98.000	103.000
	134.000	135.000	137.000	138.000	140.000	197.000	198.000	199.000	208.000
	248.000	259.000	260.000	262.000	263.000	268.000	269.000	271.000	281.000
	291.000	296.000	297.000	299.000	300.000	303.000	304.000	306.000	312.000
	313.000	315.000	320.000	322.000	324.000	370.000	371.000		
JJ	92.000	246.000	247.000	248.000	263.000	285.000	286.000	288.000	289.000
	295.000	296.000	342.000						
JK	113.000	145.000	146.000	148.000	149.000	150.000	151.000	152.000	153.000
	154.000	155.000	156.000	157.000	160.000	161.000	162.000	163.000	168.000
	169.000	171.000	172.000	174.000	175.000	177.000	178.000		
K	12.000	13.000	42.000	48.000	98.000	379.000			
KA	24.000	48.000	60.000	301.000					
KB	60.000	237.000	238.000	244.000	301.000				
KC	12.000	13.000	96.000	98.000	137.000	138.000			
KK	255.000	256.000	277.000	278.000	287.000	288.000	293.000	294.000	296.000
	297.000	305.000	306.000	314.000	315.000				
KL	24.000	48.000	235.000						
L	97.000	98.000	284.000	285.000	287.000	294.000	295.000		
LA	12.000	42.000	48.000	83.000	85.000	129.000			
LD	12.000	42.000	48.000	71.000	74.000	75.000	124.000	125.000	
LK	24.000	48.000	206.000						
LL	12.000	42.000	48.000	68.000	69.000	123.000	142.000	143.000	144.000
LN	12.000	42.000	48.000	80.000	81.000	127.000			
M	282.000	283.000	292.000	379.000					
ME	12.000	13.000	259.000	306.000	322.000	324.000			
MM	24.000	42.000	48.000	194.000					
MX	12.000	13.000	260.000	278.000	315.000	322.000	324.000		
MXIN	12.000	13.000	281.000	291.000	296.000	297.000	306.000	315.000	324.000
NL	56.000	205.000	206.000	208.000	216.000	218.000	221.000	236.000	
NN	24.000	42.000	48.000	57.000	103.000	115.000	130.000	134.000	147.000
	179.000	187.000	197.000	208.000	240.000	247.000	249.000	255.000	264.000
	269.000	270.000	276.000	277.000	280.000	282.000	285.000	287.000	291.000
	293.000	295.000	302.000	303.000	305.000	311.000	312.000	314.000	345.000
	347.000	351.000	354.000	369.000	370.000				
P	13.000	272.000	274.000	289.000	291.000	296.000	297.000		
PGPH	42.000	48.000	71.000	124.000					
PBP2	42.000	48.000							
PPT	42.000	48.000	123.000						
PT	121.000	123.000	142.000	143.000	144.000				
POP2	127.000								
RA	42.000	48.000	85.000	129.000					
RD	42.000	48.000	72.000	75.000	124.000	125.000			
RL	42.000	48.000	69.000	123.000					
RN	42.000	48.000	81.000	127.000					
SIN	121.000								
SB1	42.000	48.000	71.000	124.000					

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                              CROSS REFERENCE OF MAIN    10:19 MAR 10,'76         3
T            42.000    48.000   103.000   106.000   112.000   114.000   158.000   164.000   185.000
            208.000   216.000   218.000   221.000   375.000
TLM          42.000    48.000   375.000
W            24.000    48.000   121.000
X            13.000    42.000    48.000    58.000   103.000   106.000   116.000   154.000   160.000
            180.000   203.000   208.000   216.000   218.000   221.000
XD           13.000   188.000
XDL          13.000   189.000   253.000
XDM          13.000   191.000   242.000   252.000   253.000   266.000
XDT          13.000   190.000   242.000   252.000   266.000
XL           13.000    42.000    48.000   103.000   106.000   117.000   155.000   161.000   181.000
            198.000   208.000   218.000   221.000   251.000
XLN          13.000    58.000   103.000   106.000   203.000   208.000   218.000   221.000
XM           13.000    42.000    48.000   103.000   115.000   157.000   163.000   183.000   208.000
            221.000   241.000   250.000   251.000   265.000
XS           13.000    42.000    48.000    58.000   138.000   203.000
XT           13.000    42.000    48.000   103.000   106.000   118.000   156.000   162.000   182.000
            199.000   208.000   218.000   221.000   241.000   250.000   260.000   265.000
X2N          42.000    48.000    69.000
X4N          42.000    48.000    72.000    75.000
X6N          42.000    48.000    81.000
Y            13.000   195.000   198.000
YT           13.000   196.000   199.000
ZAP         379.000
Z1           13.000   148.000   154.000   160.000   180.000
Z2           13.000   149.000   155.000   161.000   181.000
Z3           13.000   150.000   156.000   162.000   182.000
Z4           13.000   151.000   157.000   163.000   183.000
1            47.000*   42.000
10           21.000*  380.000
11          320.000   327.000*
110          95.000    99.000*
12          322.000   328.000*
120          94.000   100.000*
13          324.000   333.000*
190         102.000   106.000*
2            25.000*   48.000
200         108.000*  376.000
3            23.000*   22.000
30           57.000    59.000*
300         115.000*  186.000
310         115.000   120.000*  167.000
315         136.000   138.000*
320         134.000   140.000*
330         130.000   141.000*
340         147.000*  170.000   173.000   176.000
350         153.000   160.000*
360         152.000   159.000   165.000*
370         147.000   166.000*
380         145.000   168.000*
390         168.000   171.000*
```

391	171.000	174.000*			
392	174.000	177.000*			
393	179.000	184.000*			
394	177.000	187.000*			
395	187.000	192.000*			
4	377.000*				
40	64.000*	63.000			
400	197.000	200.000*			
410	194.000	201.000*			
415	202.000	204.000*			
420	207.000	211.000*			
421	212.000	216.000*			
422	213.000	218.000*			
423	214.000	221.000*			
424	215.000	217.000	220.000	223.000	234.000*
480	238.000	240.000*			
485	240.000	243.000*			
490	235.000	244.000*			
5	58.000*	57.000			
50	62.000	65.000*			
500	246.000	248.000*			
510	255.000	257.000*			
520	249.000	261.000*			
530	264.000	267.000*			
6	216.000	224.000*			
600	270.000*	300.000			
610	271.000	274.000*			
620	270.000	273.000	275.000*		
630	278.000*	277.000			
635	281.000*	280.000			
640	276.000	279.000*			
650	283.000	285.000	286.000	290.000*	
658	288.000*	287.000			
660	296.000*	295.000			
670	292.000	298.000*			
7	106.000	218.000	226.000*		
700	305.000	307.000*			
710	303.000	308.000*			
720	302.000	309.000*			
755	315.000*	314.000			
756	312.000	316.000*			
757	311.000	317.000*			
758	319.000*	373.000			
7581	346.000*	345.000			
7582	348.000*	347.000			
7583	352.000*	351.000			
7584	355.000*	354.000			
7585	361.000*	360.000			
7586	365.000*	364.000			
7587	367.000*	366.000			
7588	343.000	369.000*			

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759	318.000	342.000*				
760	371.000*	370.000				
770	369.000	372.000*				
790	235.000	236.000	244.000	268.000	341.000	374.000*
8	221.000	230.000*				
800	20.000	379.000*				
9	24.000	54.000*				
99	98.000*	97.000				

APPENDIX B

SIXTH AND EIGHTH ORDER DATA SETS FOR CTL-V EXAMPLE

. ? 2 .1 ..0 .. 1. .
6 3 10 1 1C
.001,27.5,62.8,.0005,
.007,.5,.000416,500.,
.00C41,.1.4,.013,.00114,
635.,635.,.00a,.003,
.000104,635.,.056,.001,
.2,4.,300.,15.,
0.,C.,0.,C.,
133.,633.1,496.6,633.1,
41.68,633.1,0.,C.,
C.,C.,0.,C.,
0.,.2,0.,.3,
0.,.3,0.,C.,
0.,C.,C.,0.,
1200.,1598.,0.,C.,
0.,C.,0.,C.,
1477.,0.,C.,C.,
0.,C.,0.,C.,
1500.,-1306.,1.,C.,
C.,C.,0.,C.,
0.,1.,C.,C.,
0.,C.,0.,C.,
1.,C.,0.,C.,
133.,633.1,498.6,633.1,
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0.,C.,0.,C.,
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0.,C.,0.,C.,
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C.,C.,0.,0.,
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133.,633.1,498.6,633.1,
41.68,633.1,41.68,.002,
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-143.,0.,-95.3,-111.4,
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-125.,-44.03,0.,0.,
0.,0.,0.,0.,
0.,0.,0.,-990.,
0.,0.,0.,0.,
-125.,0.,17.9,-200.,
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ORIGINAL PAGE IS
OF POOR QUALITY

APPROVAL

PARAMETER ESTIMATING STATE RECONSTRUCTION

By Edwin Bruce George

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

William R. Marshall

WILLIAM R. MARSHAL
Associate Director for Engineering

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